Essays on Banking and Default

Anıl Arı

Faculty of Economics

University of Cambridge

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To Engin, Refia, Orhan and Monica

Excerpt from Ithaka

Keep Ithaka always in your mind.
Arriving there is what you are destined for.
But do not hurry the journey at all.
Better if it lasts for years,
so you are old by the time you reach the island,
wealthy with all you have gained on the way,
not expecting Ithaka to make you rich.

Ithaka gave you the marvelous journey.
Without her you would not have set out.
She has nothing left to give you now.

And if you find her poor, Ithaka won't have fooled you.
Wise as you will have become, so full of experience,
you will have understood by then what these Ithakas mean.

C.P. Cavafy

Declaration

This dissertation is the result of my own work and includes nothing which is the outcome of work done in collaboration except as declared in the Preface and specified in the text. It is not substantially the same as any that I have submitted, or, is being concurrently submitted for a degree or diploma or other qualification at the University of Cambridge or any other University or similar institution except as declared in the Preface and specified in the text. I further state that no substantial part of my dissertation has already been submitted, or, is being concurrently submitted for any such degree, diploma or other qualification at the University of Cambridge or any other University or similar institution except as declared in the Preface and specified in the text. It does not exceed the prescribed word limit of 60000 words.

Anil Ari
August 2017

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Preface

My thesis combines work on several topics within the fields of macro-finance and macroeconomics, developed in three chapters. The first two chapters contribute to the literature on bank risk-taking, sovereign debt crises, and macroeconomic dynamics during financial and debt crises. The third chapter focuses on the recent growth of shadow banking and its implications for market discipline on traditional commercial banks.

In the first chapter, titled *"Aggregate Risk and Bank Risk-Taking"*, I propose a general equilibrium model in which strategic interactions between banks and depositors may lead to endogenous bank fragility and a drop in investment and output. With some opacity in bank balance sheets, depositors form expectations about bank risk-taking and demand a return on bank deposits according to their risk. This creates strategic complementarities and possibly multiple equilibria: in response to an increase in funding costs, banks may optimally choose to pursue risky portfolios that undermine their solvency prospects. In a bad equilibrium, bank lending is crowded out by risky asset purchases and weak economic fundamentals lead to a banking crisis.

I show that this model has important implications for economic vulnerability to crises to policy design. The problem of multiple equilibria arises in countries with high aggregate risk and an under-capitalized banking sector. In these countries, policy interventions in support of the banking sector face a trade-off between alleviating banks' funding conditions and strengthening their risk-taking incentives. Due to this trade-off, liquidity provision to banks may backfire and eliminate the good equilibrium when it is not targeted. Targeted interventions have the capacity to eliminate the bad equilibrium.

In the second chapter, titled *"Gambling Traps"*, I analyze macroeconomic dynamics associated with this framework in a dynamic general equilibrium model. I show that strategic interactions between banks and depositors may leave countries stuck in "gambling traps" after adverse shocks. In a gambling trap, high bank funding costs hinder the accumulation of bank net worth, leading to a prolonged period of financial fragility and an endogenously persistent decline in economic activity.

I bring this model to bear on the European sovereign debt crisis, in the course of which under-capitalized banks in default-risky countries experienced an increase in funding costs and raised their holdings of domestic government debt. The model is quantified using Portuguese data and accounts for macroeconomic dynamics in Portugal in 2010-2016. Finally, I show that subsidized loans to banks, similar to the European Central Bank's longer-term refinancing operations (LTRO) lead to a rise in banks' holdings of risky domestic government debt and perpetuate gambling traps.

The third chapter, titled *"Shadow Banking and Market Discipline* on *Traditional Banks"*, is joint work with Matthieu Darracq-Paries, Christoffer Kok, and Dawid Żochowski. In this chapter, we propose a general equilibrium banking model in which shadow banking arises endogenously and undermines market discipline on traditional banks. We show that depositors' ability to re-optimize in response to crises imposes market discipline on traditional banks: these banks optimally commit to a safe portfolio strategy to prevent early withdrawals. When commitment is costly, shadow banking emerges as an alternative banking strategy that combines high risk-taking with early liquidation in times of crisis. We derive an equilibrium in which the shadow banking sector expands to a size where its liquidation causes a fire-sale and exposes traditional banks to liquidity risk. Higher deposit rates in compensation for liquidity risk also weaken threats of early withdrawal and traditional banks pursue risky portfolios that may leave them in default.

This theoretical model accounts for two key empirical facts about the 2007-2009 financial crisis in the United States: Shadow banks faced a sudden contraction in funding and the liquidation of their assets caused a fire-sale. Traditional banks did not suffer from withdrawals, experienced a sharp rise in their funding costs, and re-allocated their portfolios towards safe and liquid assets. The model also yields novel and important insights for policy design. We find that policy interventions aimed at alleviating fire-sales fuel further expansion of shadow banking. Financial stability can be achieved with a tax on shadow bank profits or collateralized liquidity support to traditional banks.

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Chapter 1

Aggregate Risk and Bank Risk-Taking

Abstract

I propose a general equilibrium model in which strategic interactions between banks and depositors may lead to endogenous bank fragility and decline in investment and output. With some opacity in bank balance sheets, depositors form expectations about bank risk-taking and demand a return on bank deposits according to their risk. This creates strategic complementarities and possibly multiple equilibria: in response to an increase in funding costs, banks may optimally choose to pursue risky portfolios that undermine their solvency prospects. In a bad equilibrium, bank lending is crowded out by risky asset purchases and weak fundamentals lead to a banking crisis. Policy interventions face a trade-off between alleviating banks' funding conditions and strengthening risk-taking incentives. Liquidity provision to banks may eliminate the good equilibrium when not targeted. Targeted interventions have the capacity to eliminate adverse equilibria.

Keywords: Risk taking; Banking crises; Bank regulation; Financial Constraints
JEL codes: E44, E58, F30, G11, G21, G28

1.1 Introduction

Evidence from recent financial and debt crises shows that in response to higher aggregate risk, under-capitalized banks increase their exposure to aggregate risky assets and experience a rise in their funding costs. This leads to rising bank fragility and default risk, and raises two important questions. First, what are the circumstances and mechanism that drive banks to become excessively exposed to aggregate risk? Second, what is the role of bank funding costs?

In this chapter, I propose a framework where deposits are assets priced according to their risk, and banks can optimally choose to pursue risky portfolios (which may lead to default in equilibrium) under limited liability. This creates strategic complementarities: high required deposit interest rates in anticipation of risk-taking behaviour raise the costs of funding for banks and strengthen their incentives to take on more risk. Banks may then endogenously validate depositor expectations in equilibrium, raising the possibility of multiple equilibria.

I develop my analysis by specifying a small open economy model with households, firms, entrepreneurs, and a banking sector. Banks collect deposits from households and choose their portfolios of aggregate-risky assets and loans to firms; households lend to banks on terms that depend on bank solvency prospects; entrepreneurs sell assets backed by a pool of risky projects; firms invest.

Modelling the equilibrium adjustment in bank risk-taking strategies in response to funding conditions has key macroeconomic and policy implications. The kernel intuition is that, when banks are well capitalized and/or market sentiment is "good", the resulting banking equilibrium can be described as safe. In a "safe equilibrium" banks keep their exposure to aggregate risk low. Since banks are safe, depositors accept low interest rates. With some opacity preventing depositors from observing the bank portfolio in detail, however, another equilibrium may emerge depending on the conditions of the economy and the net worth of banks. In this "gambling equilibrium", depositors expect banks to have a high exposure to aggregate risk and hence become risky themselves. As depositors require a risk premium, banks find it optimal to gamble and buy risky assets. The possibility of multiple equilibria depends on bank capitalization: the problem plagues economies where the banking sector is under-capitalized.

The model naturally provides novel and important insights on the effectiveness of central banks' liquidity interventions in support of financial intermediaries. A key prerequisite for successful interventions is that they need to provide some risk-sharing with depositors. I show that when the repayment of official debt takes precedence over deposits, liquidity provision is completely ineffective. This is because depositors anticipate the dilution of their claims to bank revenues in the event of insolvency, and raise deposit rates accordingly. The second requirement for a successful intervention is that it must be well-targeted. Non-targeted interventions that provide liquidity unconditionally face an adverse trade-off between their goals of alleviating banks' funding conditions and strengthening their incentives to gamble. When bank net worth is low, non-targeted liquidity provision may actually eliminate the safe equilibrium. In the gambling equilibrium, banks use the additional funding to increase their exposure to aggregate risk until their funding costs return to the pre-intervention level. On the contrary, targeted interventions that provide liquidity conditional on bank leverage overcome the adverse trade-off and eliminate the gambling equilibrium.

These insights can be generalized to a large set of policy instruments. I show that, on its own, deposit insurance faces the same trade-off as non-targeted liquidity provision (with risk sharing). A wide range of macroprudential policy instruments can be used in conjunction with deposit insurance to overcome the trade-off, leading to a similar outcome as targeted liquidity provision. Specifically, this outcome is implementable using regulatory constraints on bank liabilities or risk-weighted capital regulation.

In greater detail, I model the optimal strategies of banks and households as follows. When there is uncertainty about future economic fundamentals, bank managers adopt either a "safe" or a "gambling" strategy. The safe strategy consists of investing in a precautionary manner with the goal of remaining solvent even when fundamentals turn out to be weak. The gambling strategy consists of pursuing high exposure to risky assets, and leads to insolvency under adverse fundamentals.

Bank managers have incentives to gamble for two reasons: First, they are protected by limited liability. If fundamentals turn out to be strong ex-post, risky assets pay a high return driven by the risk premium; when fundamentals are weak, banks are shielded from the full consequences of their losses by limited liability. Second, they are subject to aggregate (i.e. non-diversifiable) risk. Bank managers anticipate (quantitatively small) costs that may hit them in the event of weak fundamentals independent of their holdings of risky assets. I model these costs as reflecting all balance sheet losses that a macroeconomic recession can impose on banks other than the direct impact of losses on risky assets. By way of example, a downturn usually leads to a deterioration in the value of illiquid assets and a rise in non-performing loans.

To the extent that deposit insurance is incomplete and/or lacks credibility, households optimally act on their assessment of bank solvency prospects by demanding higher rates on their deposits.[1] I first show that the dependence of bank solvency on deposit repayment obligations creates a kink in the optimal deposit schedule. Above a threshold level of deposits, households anticipate that banks will become insolvent in the event of weak fundamentals and demand higher interest payments in compensation. Another determinant of banks' solvency prospects is their exposure to risky assets. The higher this exposure is, the lower the level of deposits at

[1] Deposit insurance schemes typically guarantee deposits only up to a limit (Demirguc-Kunt et al., 2008). In real terms, depositor losses can take the form of a suspension of convertibility and a currency devaluation as well as an explicit bail-in.

which banks become insolvent in case of default. Increasing exposure thus translates into an inward shift of the deposit threshold.

With full transparency of bank balance sheets, the anticipated tightening of the deposit threshold would deter banks from increasing their exposure to aggregate risk, and by extension, rule out a gambling strategy. However, banks are typically able to obscure the composition of their investment in a variety of ways, including reliance on shell corporations and complex financial instruments.[2] I assume, realistically, that households cannot directly observe portfolio exposures and have to form expectations about banks' strategies.

I refer to anticipations of a safe strategy as "good sentiments", as opposed to "bad sentiments" associated with anticipation of gambling. Since the gambling strategy revolves around higher exposure to aggregate risk, bad sentiments result in a tightening of the deposit threshold. Bank managers strive to satisfy a solvency constraint under the safe strategy. Any shift to bad sentiments further constrains their ability to raise funds and reduces the value of the safe strategy relative to gambling. Bad sentiments may then become self-fulfilling when the tightening of the deposit threshold makes it optimal for banks to adopt the gambling strategy.

I solve for a rational expectations equilibrium and find that the characterization of the equilibrium outcome is contingent on bank net worth. With sufficiently high net worth, bank managers adopt a safe strategy regardless of the location of the deposit threshold and only positive sentiments are confirmed in equilibrium. In this "safe equilibrium", banks reduce risky asset purchases and deleverage to satisfy their solvency constraints, while bank funding costs remain at the risk-free rate. Conversely, only a gambling equilibrium may be sustained with sufficiently low net worth. In a gambling equilibrium, banks increase their exposure to risky assets at the expense of credit to firms and bank funding costs are high. Finally, sentiments become self-fulfilling in an intermediate region of net worth. A rise in aggregate risk amplifies the impact of sentiments on bank funding costs and expands this "multiplicity region".

Relationship to the literature This chapter is related to the literature on bank risk-taking. The insight that limited liability and portfolio opacity lead to risk-shifting may be traced back to Jensen and Meckling (1976) and Kareken and Wallace (1978). Krasa and Villamil (1992) emphasize the importance of aggregate risk by showing that banks take excessive risk only when they are not able to fully diversify their portfolios. It is the combination of these three ingredients that leads to the emergence of the gambling equilibrium in this chapter.

A strand of this literature focuses on competition between banks. Keeley (1990), Allen and Gale (2000) and Hellmann et al. (2000) among many others[3] develop models where imperfectly

[2] The level of deposits is public information. Although banks may also raise funds through less transparent methods, this has no impact on the repayment prospects of depositors due to their seniority.

[3] See also Marcus (1984), Suarez (1994) and Matutes and Vives (2000). Carletti (2008) provides an extensive review of this literature.

competitive banks make excess profits. Expected future profits then create skin in the game and temper banks' risk-taking incentives. In these studies, greater competition for deposits raises bank funding costs and reduces profit margins, leading to a rise in bank risk-taking. Chan et al. (1986), Besanko and Thakor (1993) and Marquez (2002) reach similar conclusions in environments where excess profits stem from informational rents. Dell'Ariccia et al. (2014) propose a model where interest rate hikes lead to a rise in bank risk-taking through a similar mechanism when banks are under-capitalized.

Boyd and De Nicolo (2005) show that introducing moral hazard in loan markets may reverse these findings. They propose a model where increased competition among banks reduces interest rates on loans. With lower loan rates, borrowers invest in less risky projects which in turn increase asset quality and reduce bank risk. Martinez-Miera and Repullo (2010) find that allowing for imperfect correlation across borrowers' default probabilities weakens this channel.

This chapter also proposes a model with imperfect competition where a rise in the funding costs of banks reduces their profit margins and increases risk-taking incentives. Different to the literature on bank competition, however, it focuses on depositor expectations about bank risk-taking as a determinant of bank funding costs. The main contribution of this chapter is to show that these expectations may become self-fulfilling: When depositors expect high risk-taking by banks, they demand a risk premium on deposits. High funding costs in turn strengthen banks' risk-taking incentives. With sufficiently low bank net worth and high aggregate risk, depositor expectations become self-fulfilling such that there are multiple equilibria.

The multiplicity mechanism considered in this chapter differs from bank-runs à la Diamond and Dybvig (1983) in that it pertains to banks' ex-ante risk-taking decisions rather than ex-post withdrawals. In the gambling equilibrium, banks invest in risky portfolios that leave them vulnerable to default risk. However, these risks only come to pass when economic fundamentals turn out to be weak. With strong fundamentals, banks remain solvent regardless of the equilibrium type.

Repullo (2004), Farhi and Tirole (2012) and Acharya et al. (2016) also propose models with multiplicity in bank risk-taking. In Repullo (2004), banks compete for deposits in a circular road model and there are multiple equilibria when the degree of competition is sufficiently strong. In Farhi and Tirole (2012) and Acharya et al. (2016), multiplicity stems from time inconsistency in the government's bailout incentives. In a bad equilibrium, banks invest in a risky asset in anticipation of government support and the correlation in their exposures makes it optimal for the government to provide support. In these studies, multiple equilibria arise due to strategic complementarities across banks. In contrast, this chapter analyzes strategic complementarities between the optimal strategies of banks and depositors.

Finally, this chapter is related to the literature on the risk-taking implications of policy interventions in support of the banking sector. Many of the studies discussed above emphasize that policy interventions face a trade-off between increasing bank profits under a safe portfolio strategy and providing additional opportunities for risk shifting. Cordella and Yeyati (2003) find that the former effect dominates when interventions are contingent on adverse macroeconomic conditions. Marquez (2017) reach a similar conclusion when a large portion of bank investors may not observe bank behaviour. In this chapter, the latter effect dominates such that deposit insurance guarantees and unconditional liquidity provision backfire and eliminate the safe equilibrium. Instead, the gambling equilibrium may be eliminated with targeted interventions which provide liquidity conditional on bank leverage.

Layout The remainder of this chapter is structured as follows: Section 1.2 describes the model environment. Section 1.3 presents the equilibrium solution. Section 1.4 conducts policy analysis. Section 1.5 concludes.

1.2 Model environment

I consider a stylized model of small open financial economy with four agents: households, banks, entrepreneurs and firms. Events unfold over two time periods (see Figure 1.1 for a graphical timeline). In the first period, banks collect deposits from households and use these funds, along with their own net worth, for asset purchases and working capital lending to firms. Entrepreneurs issue assets backed by their investment in a pool of risky projects. Firms produce the consumption good.

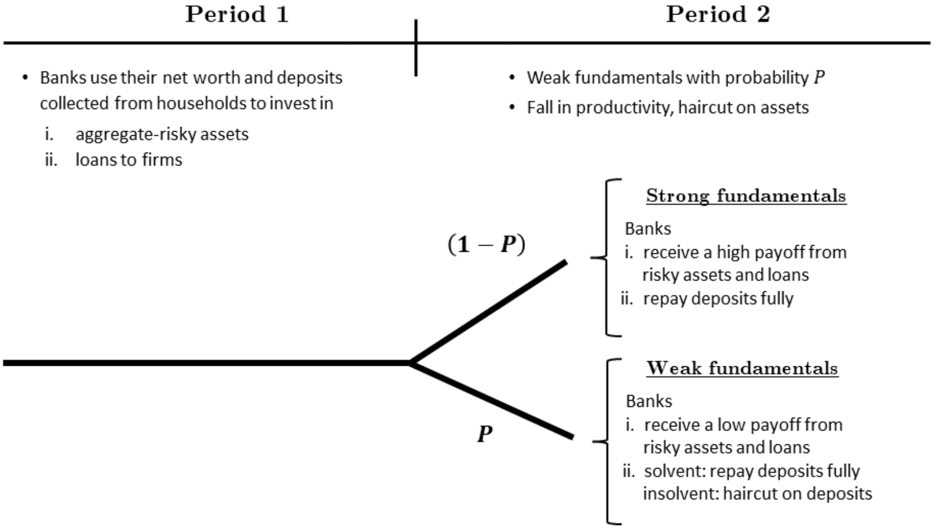
In the second period, economic fundamentals turn out to be weak with (exogenous) probability P. Weak fundamentals lead to a haircut on risky assets and reduce the productivity of firms. If banks are left with insufficient funds to pay the promised return to their depositors, they become insolvent under limited liability and a haircut proportionate to their funding shortfall is imposed on deposits.[4]

Banks' solvency prospects in the event of weak fundamentals are determined by the strategy their managers adopt in the first period. The 'safe strategy' consists of investing in a precautionary manner that leaves them solvent under weak fundamentals, whereas the 'gambling strategy' leads to insolvency. Bank managers find it optimal to follow the strategy that maximizes their expected payoff.

A key friction in the model is the limited transparency of bank balance sheets. Specifically, households observe the amount of deposits collected by banks but not their exposure to risky assets, which can be obscured through the use of shell corporations and/or complex financial

[4]The absence of safe assets among banks' investment opportunities serves only to simplify the exposition. Their inclusion would be completely inconsequential in this set up as purchasing a safe asset is either equivalent to or less profitable than a reduction in deposits by the same amount.

Figure 1.1: **Timeline**



instruments. This leads to a two-way relationship between the optimal strategies of bank managers and households. When households anticipate that banks follow a gambling strategy, their optimal deposit schedule changes in a manner that increases banks' incentives to gamble. Household expectations about bank risk-taking may then become self-fulfilling.

Finally, before I explain these activities in more detail, it is convenient to describe some notational conventions. Table 1.1 provides a list of variables and parameters. Deposits, risky assets, loans and safe assets are respectively labelled as (d, b, l, d^*) and take the form of discount bonds with prices $\left(q, q^b, q^l, q^*\right)$.[5] The recovery rates of (d, b, l) under weak fundamentals are $\left(\theta, \theta^b, \theta^l\right)$. An underbar denotes variables at the state with weak fundamentals such that $\underline{A}$ is productivity when fundamentals turn out to be weak. Aggregate quantities, such as aggregate loans L, are in the upper case while lower case variables pertain to an individual bank.

1.2.1 Entrepreneurs

In the first period, entrepreneurs invest in a large pool of high-risk projects and issue assets b backed by their returns. Diversification across projects ensures that asset payoffs are contingent only on the aggregate state such that they pay a recovery rate $\theta^b \in (0, 1)$ when fundamentals turn out to be weak.[6]

Assets are internationally traded and their marginal buyers are deep pocketed foreign in-

[5] This helps simplify the exposition without any actual impact on the model mechanisms.

[6] Note that these assets may be interpreted as any asset class with high aggregate risk. For example, they could take the interpretation of mortgage-backed securities in the context of the 2007-09 financial crisis in the United States, sovereign bonds issued by periphery countries during the Eurocrisis, or any asset denominated in domestic currency in a currency crisis.

Table 1.1: **Notation**

Variables

Label	Description
d	Deposits
b	Aggregate-risky assets
l	Loans to firms
d^*	Safe assets
q, q^l, q^b, q^*	Asset prices
$\theta, \theta^l, \theta^b$	Recovery rates
H	Labour supply
w	Wages
K	Working capital
Y	Output
n	Bank net worth
π	Bank profits
v	Bank expected payoff
γ	Exposure to aggregate risk
c	Consumption
μ_l	Loans market mark-up
μ_d	Deposit market mark-up

Parameters

Label	Description
P	Prob. of weak fundamentals
ϕ	Market share of banks
A	Productivity
α	Cobb-Douglas elasticity
β	Discount factor
$\bar{E}$	Household endowment

vestors. As such, they are priced at their expected return

$$q^b = \left(1 - P + P\theta^b\right) q^* \tag{1.1}$$

where $1/q^*$ is the risk-free rate.[7]

1.2.2 Firms

Firms are perfectly competitive. In order to produce the consumption good Y, they hire labour H from households at a wage w and borrow working capital

$$K = q^l L \tag{1.2}$$

[7] I implicitly assume that the availability of risky projects is large relative to the size of the domestic banking sector. In a monetary union setting, $1/q^*$ can be interpreted as the interest rate set by the common central bank.

from the domestic banking sector. In the interest of a clear exposition, loans to firms take the form of discount bonds L sold at a price q^l. Under a standard Cobb-Douglas production function, the representative firm's profit maximization problem is

$$\max_{K,L,H,\underline{H}} \quad (1-P)\left[AK^{\alpha}H^{1-\alpha} - L - wH\right] + P\left[\underline{A}K^{\alpha}\underline{H}^{1-\alpha} - \theta^l L - \underline{w}\underline{H}\right]$$

subject to (1.2), where A is productivity and θ^l is the recovery rate of loans. Crucially, $\left(q^l, L, K\right)$ are not state contingent as firms borrow in advance. When fundamentals are weak, loans become non-performing due to the productivity decline $\underline{A} < A$ and banks claim the firm's revenues net of salary payments such that[8]

$$\theta^l = \frac{\underline{A}K^{\alpha}\underline{H}^{1-\alpha} - \underline{w}\underline{H}}{L}$$

Combining this with the first order conditions of the firm's problem yields the expressions

$$\begin{aligned} w &= (1-\alpha)\, AK^{\alpha} \\ \underline{w} &= (1-\alpha)\, \underline{A}K^{\alpha} \\ q^l &= \left(\frac{1}{\alpha A}\right)^{\frac{1}{\alpha}} L^{\frac{1-\alpha}{\alpha}} \\ \theta^l &= \frac{\underline{A}}{A} \end{aligned} \tag{1.3}$$

where labour supply is perfectly inelastic and normalized to $H = \underline{H} = 1$. Of particular importance are the last two expressions, which respectively establish an upward-sloping loan supply schedule and pin down the recovery rate.

1.2.3 Households

There is a unit continuum of risk neutral households with an initial endowment $\bar{E}$. They save by purchasing safe assets D^* at a price q^* or deposits D from domestic banks at a price q.[9,10]

[8] This is the reduced-form outcome of a re-negotiation game between firms and banks after loans become non-performing. As firms are perfectly competitive and banks have market power, the latter extracts all of the remaining revenues after salary payments. Implicitly, this relies on the absence of information asymmetries, which can be motivated by relationship banking. This also makes it prohibitively costly for households and foreign entities to lend directly to firms. The domestic banking sector thus acts as a financial intermediary that channels funds to firms. Note that the outcome here is equivalent to the issuance of state-contingent debt by firms.

[9] The assumption of risk neutrality only serves to attain a tractable expression for the deposit demand schedule. The results presented below retain their validity under risk aversion, which is introduced in a similar model in Chapter 2. Similarly, permitting households to purchase risky assets has no effect on the outcome.

[10] D^* can be interpreted as deposits in a safe foreign bank or simply as a safe real asset. As there is a unit continuum of homogenous households, individual households' deposits are identical to the aggregate quantities. I abuse notation by using the aggregate terms (D, D^*) to describe the household's problem.

The representative household's utility maximization problem can be described as follows

$$\max_{c_1, c_2, \underline{c}_2, D, D^*} u(c_1) + \beta\left[(1-P)\, u(c_2) + Pu(\underline{c}_2)\right]$$

subject to the period budget constraints

$$c_1 + qD + q^* D^* = \bar{E}$$
$$c_2 = D + D^* + w$$
$$\underline{c}_2 = \theta D + D^* + \underline{w}$$

where β is the rate at which households discount future consumption and θ is the recovery rate of domestic bank deposits under weak fundamentals. This yields the first order conditions

$$q^* = \beta \tag{1.4}$$
$$q = (1 - P + P\theta)\, q^* \tag{1.5}$$

which indicate that domestic deposits are priced at their expected return relative to the safe asset. Observe that households' valuation of domestic deposits increases in recovery rate θ. I provide an expression for θ in the next section before deriving the optimal deposit demand schedule of households in Section 1.2.5.

1.2.4 Banks

The domestic banking sector is imperfectly competitive in the manner of Cournot. Each bank is risk neutral with a market share $\phi \in (0, 1]$. The representative bank finances its risky asset purchases and lending to firms with deposits collected from households as well as its own net worth $n \geq 0$. Its budget constraint can be written as

$$n + qd = q^b b + q^l l \tag{1.6}$$

where $l = \phi L$, $d = \phi D$ represent lending and deposits at individual bank level. Profits are contingent on economic fundamentals as follows

$$\pi = \max\{0, l + b - d\} \tag{1.7}$$
$$\underline{\pi} = \max\left\{0, \theta^l l + \theta^b b - d\right\} \tag{1.8}$$

where $\underline{\pi}$ represents profits in the event of weak fundamentals, and the maximum operators reflect limited liability. Banks always make a strictly positive profit under strong fundamentals

($\pi > 0$) but may become reliant on limited liability after fundamentals turn out to be weak. This leads to insolvency, with losses passed on to depositors through a haircut on deposits. The recovery rate of deposits reflects the bank's shortfall of funds[11]

$$\theta = \min\left\{1, \frac{\theta^l l + \theta^b b}{d}\right\} \tag{1.9}$$

with $\theta < 1$ indicating that limited liability binds.

The representative bank chooses its deposits d, asset purchases b and loans l in order to maximize its expected payoff

$$v = (1-P)\,\pi + P\underline{\pi}$$

subject to the budget constraint. Note that choosing (b, l) is equivalent to selecting the share of funds $\gamma \in [0,1]$ spent on risky asset purchases, which I refer to as banks' exposure to aggregate risk. Using (1.6), (b, l) can be defined in terms of γ as

$$b = \gamma\left(\frac{n+qd}{q^b}\right) \tag{1.10}$$

$$l = (1-\gamma)\left(\frac{n+qd}{q^l}\right) \tag{1.11}$$

It is convenient for the remainder of the text to express the recovery rate θ in terms of aggregate risk exposure γ

$$\theta = \begin{cases} 1 & \text{for } d \leq \bar{d}\,(\gamma) \\ \left(\gamma\frac{\theta^b}{q^b} + (1-\gamma)\frac{\theta^l}{q^l}\right)\left(\frac{n}{d}+q\right) & \text{for } d > \bar{d}\,(\gamma) \end{cases} \tag{1.12}$$

$$\bar{d}\,(\gamma) = \frac{\left(\gamma\frac{\theta^b}{q^b} + (1-\gamma)\frac{\theta^l}{q^l}\right)n}{1-q^*\left(\gamma\frac{\theta^b}{q^b} + (1-\gamma)\frac{\theta^l}{q^l}\right)} \tag{1.13}$$

where $\bar{d}\,(\gamma)$ represents the threshold of deposits above which the bank becomes insolvent under weak fundamentals.[12] Observe that $\bar{d}\,(\gamma)$ and θ are positively related to bank net worth n and the rate of return $\gamma\frac{\theta^b}{q^b} + (1-\gamma)\frac{\theta^l}{q^l}$ on bank funds.

Recall from the previous section that the price of deposits q increases in θ. Under imperfect competition, banks internalize the effects of their actions on θ and hence q. As such, it is necessary to determine the household's optimal deposit demand schedule in the next section before evaluating bank strategies in Section 1.2.6.

[11] There is no deposit insurance or bailot guarantees in the baseline model. These are evaluated as policy interventions in Section 1.4.

[12] This can also be interpreted as a leverage threshold $\bar{d}\,(\gamma)/n$. The claim that $\theta < 1$ for $d > \bar{d}\,(\gamma)$ is valid under the parameter restrictions discussed in the next section.

1.2.5 Deposit demand schedule

Combining (1.5) with (1.12) yields the household's optimal deposit demand schedule contingent on γ

$$q(\gamma, d) = \begin{cases} q^* & \text{for } d \leq \bar{d}(\gamma) \\ q^* \frac{1-P+P\left(\gamma \frac{\theta^b}{q^b}+(1-\gamma)\frac{\theta^l}{q^l}\right)\frac{n}{d}}{1-q^* P\left(\gamma\frac{\theta^b}{q^b}+(1-\gamma)\frac{\theta^l}{q^l}\right)} & \text{for } d > \bar{d}(\gamma) \end{cases} \tag{1.14}$$

where $\bar{d}(\gamma)$ is defined by (1.13). The deposit demand schedule is downward sloping and negatively related to γ under the parameter restrictions

$$\frac{\alpha(1-P)}{\alpha(1-P)+\phi(1-\alpha)} > \frac{\underline{A}}{\bar{A}} > \frac{\alpha\theta^b}{\alpha+\phi(1-\alpha)} \tag{1.15}$$

These restrictions ensure that following a realization of weak economic fundamentals, the rate of return from lending to firms falls short of the promised return on deposits but exceeds that of risky assets. When the first inequality is satisfied, the bank becomes insolvent under weak fundamentals given $d > \bar{d}(\gamma)$ and the deposit demand schedule is downward sloping in this region. Therefore, I refer to $d > \bar{d}(\gamma)$ as the 'risky' region of the deposit demand schedule and $d \leq \bar{d}(\gamma)$ as the 'safe region'. In the safe region, deposits are deemed to be risk-free with $\theta = 1$ by households and priced on par with safe assets $q = q^*$. Conversely, in the risky region, households price deposits at a discount $q < q^*$ in anticipation of a haircut $(\theta < 1)$ when fundamentals turn out to be weak. At the limit $d \to \infty$, the recovery rate tends to the rate of return on bank funds and the value of deposits approaches the lower bound

$$\lim_{d\to\infty} q(\gamma, d) = q^* \frac{1-P}{1-q^*P\left(\gamma\frac{\theta^b}{q^b}+(1-\gamma)\frac{\theta^l}{q^l}\right)}$$

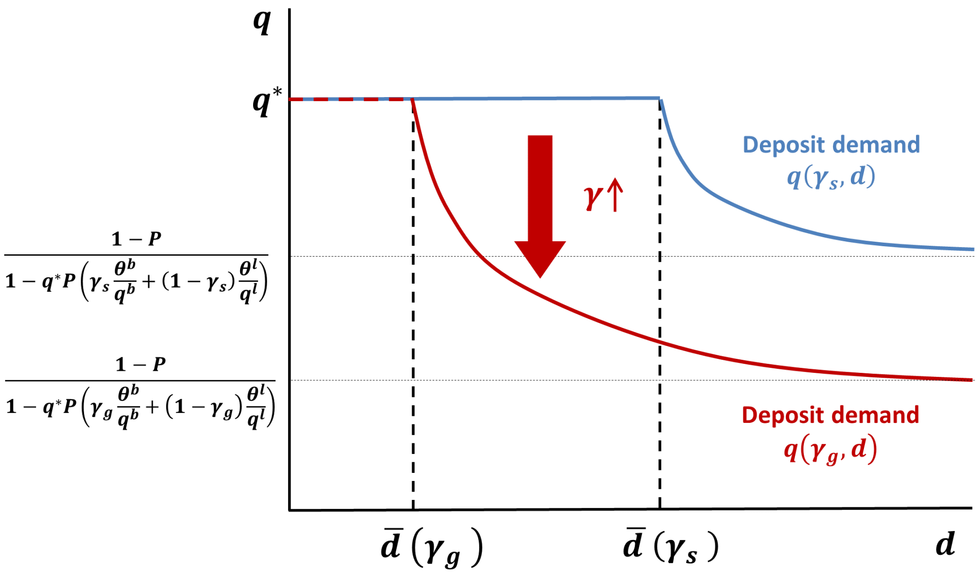
The second inequality in (1.15) establishes a negative relationship between the exposure to aggregate risk γ and the rate of return on bank funds. This ensures that the deposit threshold $\bar{d}(\gamma)$ shifts inwards in response to a rise in γ, while the risky region of the deposit demand schedule pivots downward. Figure 1.2 shows the effect of a rise in aggregate risk exposure from an arbitrary level γ_s to $\gamma_g > \gamma_s$ on the deposit demand schedule.

When bank balance sheets are completely transparent, bank managers internalize the negative relationship between exposure to aggregate risk and their funding conditions. Lemma 1.1 shows that this imposes market discipline and deters banks from gambling on risky assets.

Lemma 1.1 *When households can observe both (d, γ), limited liability has no impact on banks' optimal strategy.*

Proof. Provided in Appendix A3.1. ■

Figure 1.2: **Deposit demand schedule**



Along with the parameter restrictions, a necessary assumption to attain the results described below is some opacity in bank balance sheets such that households can observe the amount of deposits d collected by banks but not the aggregate risk exposure γ. As a result, banks cannot commit to a certain level of exposure.[13]

I elaborate further on the formation of household expectations on γ in Section 1.3.2. This discussion builds upon optimal bank strategies, however, which necessitates their explanation in advance. In the meantime, both the deposit demand schedule and the bank strategies described in the next section should be taken to be contingent on household expectations about exposure to aggregate risk, which I label as $\tilde{\gamma}$. Lacking commitment, banks take $\tilde{\gamma}$ as given and do not internalize the impact of their exposure on the deposit demand schedule $q\left(\tilde{\gamma}, d\right)$ facing them.

1.2.6 Bank strategies

Limited liability creates a discontinuity in the representative bank's optimal strategy such that it can be evaluated as a choice between two distinct strategies. Under a 'safe strategy' (labelled as 's'), the bank satisfies a solvency constraint

$$d \leq \theta^l l + \theta^b b \tag{1.16}$$

which ensures that it does not rely on limited liability when fundamentals turn out to be weak. The 'gambling strategy' (labelled as 'g'), on the other hand, results in the bank's insolvency

[13] The same outcome can be attained with a timing friction whereby banks collect deposits first and then determine their exposure γ.

and the imposition of a haircut on deposits under weak fundamentals.

In the first period, the representative bank adopts the strategy that maximizes its expected payoff such that the safe strategy is preferred when

$$v_s \geq v_g$$

where (v_s, v_g) are respectively the expected payoffs associated with safe and gambling strategies.

Gambling strategy When the bank follows the gambling strategy, it solves the problem

$$\begin{aligned} v_g &= \max_{d,\gamma\in[0,1]} (1-P)(l+b-d) \\ & \text{s.t.} \\ n+qd &= q^b b + q^l l \end{aligned} \tag{1.17}$$

where (1.10) and (1.11) map the choice of γ into (b, l). Since limited liability binds under weak fundamentals, the bank only internalizes the payoff in the state with strong fundamentals. It also internalizes the deposit demand and loan supply schedules

$$q \equiv q(\tilde{\gamma}, d) \tag{1.18}$$

$$q^l = \left(\frac{1}{\alpha A}\right)^{\frac{1}{\alpha}} (l + (1-\phi) L)^{\frac{1-\alpha}{\alpha}} \tag{1.19}$$

given by (1.14) and (1.3) due to imperfect competition.[14]

The first order conditions can then be written as

$$q^b = (1-\mu_d(\tilde{\gamma}, d))\, q \tag{1.20}$$

$$q^l = (1-\mu_l)\, q^b \tag{1.21}$$

where $\mu_d(\tilde{\gamma}, d)$ and μ_l are the mark-ups the bank enjoys in the deposit and loan markets due

[14] (1.19) differs slightly from (1.3) as it is from the perspective of an individual bank. L represents aggregate bank lending which is taken as given by the representative bank.

to its market power. They are defined as[15]

$$\mu_d(\tilde{\gamma}, d) \equiv -\frac{\partial q(\tilde{\gamma}, d)}{\partial d}\frac{d}{q} = \begin{cases} 0 & \text{for } d \leq \bar{d}(\tilde{\gamma}) \\ \frac{P\left(\tilde{\gamma}\frac{\theta^b}{q^b}+(1-\tilde{\gamma})\frac{\theta^l}{q^l}\right)\frac{n}{d}}{1-P+P\left(\tilde{\gamma}\frac{\theta^b}{q^b}+(1-\tilde{\gamma})\frac{\theta^l}{q^l}\right)\frac{n}{d}} & \text{for } d > \bar{d}(\tilde{\gamma}) \end{cases} \tag{1.22}$$

$$\mu_l \equiv \frac{\phi(1-\alpha)}{\alpha+\phi(1-\alpha)} \tag{1.23}$$

Observe that the recovery rates $\left(\theta^b, \theta^l\right)$ do not feature in the first order conditions, since the bank does not internalize its payoff under weak fundamentals. I elaborate further on the consequences of this while considering the gambling equilibrium in Section 1.3.1.

Safe strategy Under the safe strategy, the bank's problem differs from its gambling counterpart in two respects. First, as the bank does not rely on limited liability, the objective function internalizes the payoff in both states of nature such that

$$\begin{aligned} v_s &= \max_{d,\gamma\in[0,1]} (1-P)\pi + P\underline{\pi} \\ &= \max_{d,\gamma\in[0,1]} (1-P)(l+b) + P\left(\theta^l l + \theta^b b\right) - d \end{aligned}$$

Second, this is subject to an occasionally binding solvency constraint given by (1.16) in addition to the budget constraint. The first order conditions for the safe strategy can then be written as

$$\left(\theta^l l + \theta^b b - d\right)\lambda = 0 \ , \ \lambda \geq 0 \ , \ d \leq \theta^l l + \theta^b b \tag{1.24}$$

$$q^b \geq \frac{\left(1-P+P\theta^b\right)+\lambda\theta^b}{1+\lambda}\left(1-\mu_d(\tilde{\gamma}, d)\right)q \tag{1.25}$$

$$q^l = \frac{\left(1-P+P\theta^l\right)+\lambda\theta^l}{1+\lambda}\left(1-\mu_l\right)\left(1-\mu_d(\tilde{\gamma}, d)\right)q \tag{1.26}$$

where λ is the Lagrange multiplier for the solvency constraint and (1.24) is the corresponding complementary slackness condition. Compared to the gambling case, the bank has a lower valuation for both b and l since it internalizes the low payoff from these assets in the state with weak economic fundamentals. When $\theta^l > \theta^b$, however, greater value is placed on loans compared to risky assets relative to the gambling case. Both of these effects are amplified when the solvency constraint is binding such that $\lambda > 0$.

The weak inequality in (1.25) reflects the possibility that the bank may prefer not to purchase

[15] Observe that there is no deposit market mark-up in the safe region of the deposit demand schedule. This is because banks face a horizontal deposit demand schedule in this region as their deposits become perfectly substitutable with safe assets.

any risky assets ($\gamma = 0$), since their price is fixed at $q^b = \left(1 - P + P\theta^b\right) q^*$ as explained in Section 1.2.1.[16] Lemma 1.2 describes the conditions under which (1.25) holds with equality.

Lemma 1.2 *When $\lambda = 0$ and $q = q^*$, condition (1.25) holds with equality and reduces to*

$$q^b = \left(1 - P + P\theta^b\right) q^* \tag{1.27}$$

and there is an interior solution for b within the range

$$b \in \left[0, \frac{q^* \bar{d}\left(\tilde{\gamma}\right) + n - q^l l}{q^b}\right] \tag{1.28}$$

Otherwise, there is a strict inequality and a corner solution

$$q^b > \frac{\left(1 - P + P\theta^b\right) + \lambda\theta^b}{1 + \lambda} \left(1 - \mu_d\left(\tilde{\gamma}, d\right)\right) q$$
$$b = 0$$

Proof. Provided in Appendix A3.2. ■

This indicates that the bank only purchases a positive amount of risky assets $b > 0$ when the solvency constraint is slack with $\lambda = 0$ and bank deposits are at the safe region of the deposit demand schedule such that $q = q^*$. In this case, (1.27) shows that the bank's valuation of risky assets is at their expected payoff, which is equivalent to their market price given by (1.1). The bank is thus indifferent to the amount of its risky asset purchases within the range (1.28). On the other hand, when the solvency constraint binds ($\lambda > 0$) and/or bank deposits are considered to be risky ($q < q^*$), the bank does not purchase any risky assets.

In the next section, I characterize two candidate equilibria and determine the conditions under which they are self-confirming.

1.3 Equilibrium

I solve for a symmetric rational expectations equilibrium which requires that all optimality conditions and constraints of banks, firms and households are satisfied, and household expectations on aggregate risk exposure $\tilde{\gamma}$ are confirmed in the equilibrium.[17] Section 1.3.1 characterizes the

[16] Implicitly, this is a complementary slackness condition for an occassionally binding non-negativity constraint $b \geq 0$. This constraint never binds under the gambling strategy due to the higher valuation of risky assets. An equivalent constraint for lending ($l \geq 0$) is also slack at all times since q^l declines in response to a fall in l.

[17] I abstain from mixed equilibria, as this would complicate the solution significantly without yielding any interesting insights in addition to those provided by analyzing symmetric equilibria. Note also that the candidate equilibria described here, and the conditions under which they are valid, would remain valid even when mixed equilibria are taken into account.

candidate equilibria. Section 1.3.2 describes how households formulate their expectations $\tilde{\gamma}$. Section 1.3.3 provides the equilibrium conditions as well as an intuitive demonstration of the mechanism behind multiple equilibria. Finally, Section 1.3.4 formally characterizes the equilibrium regions.

1.3.1 Candidate equilibria

Under rational expectations, two candidate equilibria emerge: a 'gambling equilibrium' where household expectations of high exposure to aggregate risk in the banking sector are confirmed by the adoption of a gambling strategy by banks, and a 'safe equilibrium' where the opposite is true. With a slight abuse of notation, I use the labels 'g' and 's' to refer to variables pertaining to the gambling and safe equilibria.

Gambling equilibrium Under the gambling equilibrium, banks follow the first order conditions (1.20) and (1.21). The aggregate risk exposure γ_g, which must be consistent with household expectations $\tilde{\gamma}$, is determined by combining (1.20) with the deposit demand schedule (1.14). This yields

$$\begin{aligned} \gamma_g &\rightarrow 1 \\ q_g &\rightarrow q^b \end{aligned} \tag{1.29}$$

where the main takeaway is the co-movement between the value of deposits q_g and risky asset prices q^b. Note that the corner solution in γ_g is due to the risk neutrality of households.[18] In Appendix A1, I show that risk aversion leads to an interior solution $\gamma_g \in (0,1)$, $q_g \in \left(q^b, q^*\right)$ while preserving the co-movement property.

The second condition (1.21) pins down the price and quantity of loans purchased by the representative bank as

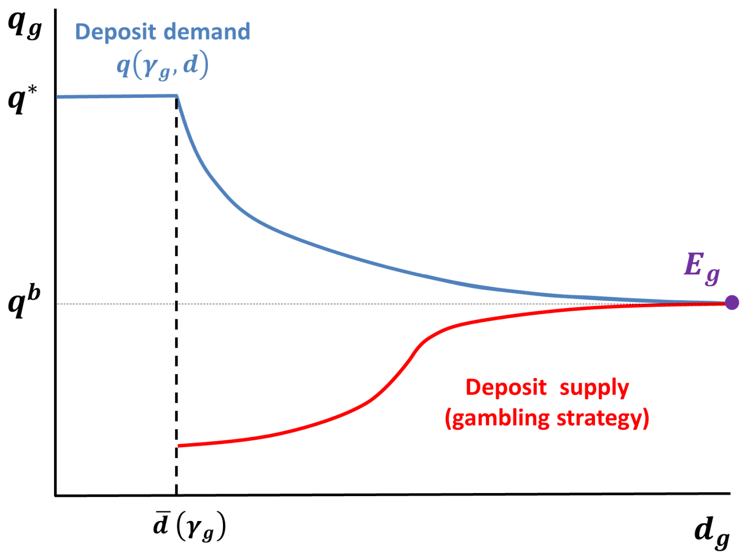
$$q^l_g = (1-\mu_l)\, q^b \tag{1.30}$$

$$l_g = \phi\,(\alpha A)^{\frac{1}{1-\alpha}}\, {q^l_g}^{\frac{\alpha}{1-\alpha}} \tag{1.31}$$

where aggregate loans are given by $L_g = l_g/\phi$. Since the bank only internalizes asset payoffs in the state with strong fundamentals, a rise in the probability P of weak fundamentals (which reduces q^b) leads to a decline in bank lending. This reflects the crowding out of bank lending

[18] Under risk neutrality, bank deposits are priced at their expected value and the curvature of the deposit demand schedule is such that the mark-up $\mu_d(\tilde{\gamma}, d)$ tends to zero as deposits increase. Therefore, under a gambling strategy, banks find it profitable to issue more deposits and use the funds to purchase risky assets until their anticipated exposure approaches unity.

Figure 1.3: **Gambling equilbrium**



Note: The deposit supply curve is attained by combining (1.19)-(1.22). Deposit demand stems from the combination of (1.14) and (1.29).

by risky asset purchases.

Finally, the expected payoff of banks under the gambling equilibrium is given by

$$v_g = (1 - P)\,\mu_l l_g + \frac{n}{q^*} \tag{1.32}$$

where the first term reflects the mark-up from lending and the second term is the expected return on banks' initial net worth.

Figure 1.3 provides a graphical depiction of the gambling equilibrium, where the red line represents the bank's optimal deposit supply schedule under a gambling strategy and E_g marks the equilibrium allocation.[19]

Safe Equilibrium Under the safe equilibrium, the deposit threshold $\bar{d}\,(\gamma_s)$ coincides with the solvency constraint (1.16) such that banks always remain within the safe region of the

[19]Observe that the rate of change in the deposit supply schedule changes direction. This occurs at $q_g = q_g^l / \left[(1-\mu_l)\,(1-\mu_d\,(\tilde{\gamma}, d))\right]$. Until this point, the bank invests only in lending to firms. By virtue of diminishing returns to scale in the production function, q^l increases at an increasing rate and so does the deposit supply schedule. Beyond this point, however, the bank invests additional funds in risky assets and the deposit supply schedule is guided by (1.20). The relationship between $\mu_d\,(\tilde{\gamma}, d)$ and d then gives the schedule a positive, but decreasing rate of change that tends to zero at $q_g \to q^b$.

deposit demand schedule with $q_s = q^*$. The first order conditions can then be written as

$$q^b \geq \frac{\left(1-P+P\theta^b\right)+\lambda\theta^b}{1+\lambda} q^* \tag{1.33}$$

$$q^l = \frac{\left(1-P+P\theta^l\right)+\lambda\theta^l}{1+\lambda}\left(1-\mu_l\right) q^* \tag{1.34}$$

It follows from Lemma 1.2 that there are two possible cases of the safe equilibrium, one where the solvency constraint is slack and another where it binds. Lemma 1.3 characterizes the safe equilibrium under both of these cases.

Lemma 1.3 *There are two cases of the safe equilibrium*

Case 1 *When $n \geq n_c \equiv \left(q_s^l - q^*\theta^l\right) l_s$, the solvency constraint is slack $(\lambda = 0)$ and (1.33) holds with equality. The safe equilibrium is then characterized by*[20]

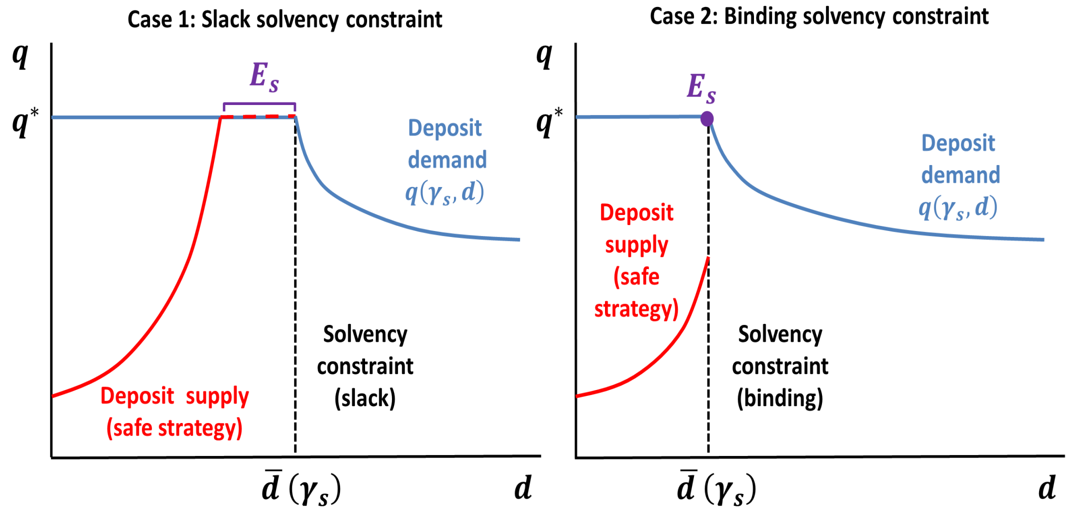
$$\begin{aligned}
q_s^l &= \left(1-P+P\theta^l\right)\left(1-\mu_l\right) q^* && (1.35)\\
l_s &= \phi\left(\alpha A\right)^{\frac{1}{1-\alpha}} q_s^{l\,\frac{\alpha}{1-\alpha}} && (1.36)\\
b_s &\in \left[0, \frac{n-\left(q_s^l - q^*\theta^l\right) l_s}{q^b - q^*\theta^b}\right] && (1.37)\\
d_s &= \frac{q^b b_s + q^l l_s - n}{q^*} &&\\
\gamma_s &= \frac{q^b b_s}{q^* d_s + n} && (1.38)\\
v_s &= \left(1-P+P\theta^l\right)\mu_l l_s + \frac{n}{q^*} && (1.39)
\end{aligned}$$

Case 2 *When $n < n_c$, the solvency constraint binds $(\lambda > 0)$ and the safe equilibrium is characterized by*

$$\begin{aligned}
q^*\theta^l l_s &= \left(\frac{1}{\phi}\right)^{\frac{1-\alpha}{\alpha}}\left(\frac{l_s}{\alpha A}\right)^{\frac{1}{\alpha}} - n && (1.40)\\
q_s^l &= \left(\frac{1}{\alpha A}\right)^{\frac{1}{\alpha}}\left(\frac{l_s}{\phi}\right)^{\frac{1-\alpha}{\alpha}} &&\\
b_s &= \gamma_s = 0 && (1.41)\\
d_s &= \theta^l l_s &&\\
v_s &= \left(1-P\right)\left(1-\theta^l\right) l_s && (1.42)
\end{aligned}$$

[20] In the definiton for n_c, $\left(q_s^l, l_s\right)$ correspond to (1.35), (1.36)

Figure 1.4: **Safe equilibrium**



Note: The deposit supply curve is attained by combining (1.3) with (1.34) and (1.34). Deposit demand stems from the combination of (1.14) and (1.38).

where the parameter restrictions (1.15) are sufficient to show that

$$\frac{\partial l_s}{\partial n} > 0 \;\forall\; n < n_c$$

Proof. Provided in Appendix A3.3. ■

Figure 1.4 represents the two cases graphically. In the first case, banks value assets according to their expected return since they do not face a binding constraint or expect to rely on limited liability. The equilibrium price of loans is then given by (1.35). As explained in section 1.2.6, banks are indifferent to the amount of their risky asset purchases within a range given by (1.37), because their valuation of these assets coincides with their market price. Consistent with this, there is also a range of admittable equilibrium values for (d_s, γ_s). In Figure 1.4, this is depicted by the overlapping region E_s between the deposit demand and supply curves. In order to pin down these variables in equilibrium, I select the upper bound of (1.37) as the equilibrium value for b_s. This amounts to eliminating a range of safe equilibria with lower (b_s, γ_s) values without any impact on the characteristics of the equilibrium outcome.[21]

In the second case, the binding solvency constraint creates a wedge between the demand and supply of deposits. Therefore, banks do not find it optimal to purchase any risky assets and the equilibrium quantity of loans is implicitly defined by (1.40). A rise in net worth n relaxes the solvency constraint, leading to a rise in the price and quantity of loans.

Finally, it is worth discussing bank lending in the context of safe and gambling equilibria.

[21]The parameter regions under which the safe equilibrium with the selected b_s value exists fully encompasses that of safe equilibria with lower b_s values. In other words, whenever the safe equilibria with lower b_s values exist, so does the selected equilibrium, which is identical to them in all other aspects.

Proposition 1.1 outlines the conditions under which a gambling equilibrium is associated with lower bank lending.

Proposition 1.1 *Bank lending is lower in a gambling equilibrium under the conditions*

$$\theta^l > \theta^b$$
$$n > \left(\frac{1}{\phi}\right)^{\frac{1-\alpha}{\alpha}} \left(\frac{l_g}{\alpha A}\right)^{\frac{1}{\alpha}} - q^* \theta^l l_g$$

Proof. Provided in Appendix A3.5. ■

The first condition pertains to banks' risk-taking incentives. In a gambling equilibrium, an adverse change in economic fundamentals drives the banking sector into insolvency. Because of limited liability, banks then cease to internalize their revenues in the state with weak fundamentals. When the recovery rate of loans exceeds that of risky assets, this leads to the crowding out of bank lending by risky asset purchases.

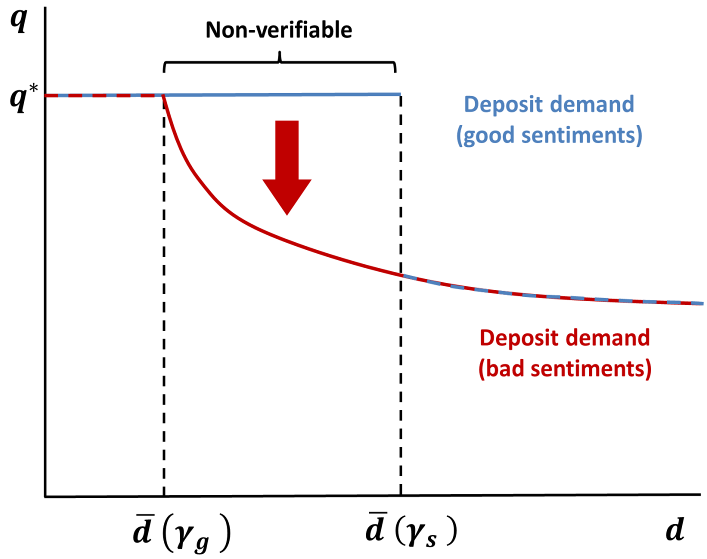
In spite of this, bank lending is higher under the gambling equilibrium when net worth falls short of the level required to satisfy the second condition. In this case, a tight solvency constraint forces banks to reduce their lending below the gambling level in order to ensure their solvency under weak fundamentals. Note that as the recovery rate θ^l of loans increases, the second condition is satisfied at a wider range of net worth, while crowding out effects get stronger.

1.3.2 Sentiments

Recall from Section 1.2.5 that banks' aggregate risk exposure γ is unobservable. Nevertheless, it is a key determinant of their solvency prospects and hence the optimal deposit demand schedule $q(\gamma, d)$. In this section, I describe how households formulate their expectations $\tilde{\gamma}$ about banks' exposures to aggregate risk. This is equivalent to forming an expectation about bank strategies since (1.29), (1.38) and (1.41) establish a one-to-one mapping between the two conditional on the observables (n, d).

Figure 1.2 shows the deposit demand schedules associated with the expectation of safe $(\tilde{\gamma} = \gamma_s)$ and gambling $(\tilde{\gamma} = \gamma_g)$ strategies. Observe that households may infer the bank strategy from the level of deposits d when it lies outside the range $d \in (\bar{d}(\gamma_g), \bar{d}(\gamma_s)]$. When $d \leq \bar{d}(\gamma_g)$, banks remain solvent under weak fundamentals even when their exposure is at a level associated with the gambling strategy. As such, banks cannot possibly follow a gambling strategy when their deposits remain within this region. Similarly, even the low exposure γ_s associated with the safe strategy leads to insolvency when deposits exceed $\bar{d}(\gamma_s)$ such that $d > \bar{d}(\gamma_s)$ is not consistent with a safe strategy.

Figure 1.5: **Sentiments**



In contrast, within the 'non-verifiable' region $d \in (\bar{d}\left(\gamma_g\right), \bar{d}\left(\gamma_s\right)]$, it is not possible to deduce the bank strategy from observables. Expectations about the aggregate risk exposure $\tilde{\gamma}$ are instead determined by household sentiments such that 'good sentiments' refer to the expectation of a safe strategy and 'bad sentiments' refer to that of a gambling strategy. Figure 1.5 displays the deposit demand schedule under each type of sentiments. As I solve for a rational expectations equilibrium, sentiments can only exist when they are self-confirming in equilibrium.

1.3.3 Equilibrium conditions

Under the rational expectations equilibrium framework described in section 1.3.1, the safe equilibrium exists when the representative bank finds it optimal to follow a safe strategy provided that there are good sentiments and other banks also follow a safe strategy. This leads to the equilibrium condition

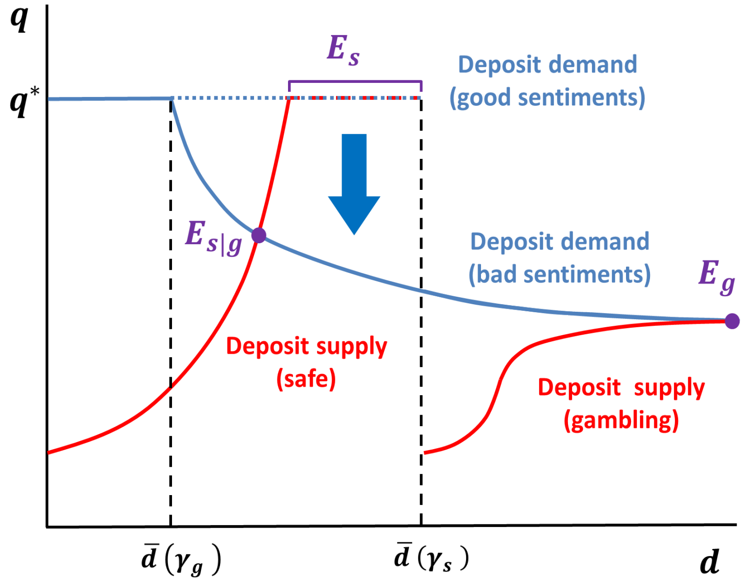
$$v_s \geq v_{g|s} \tag{1.43}$$

where v_s is the representative bank's expected payoff in the safe equilibrium given in Lemma 1.3 and $v_{g|s}$ is the expected payoff from a 'deviation to the gambling strategy'. I refer to $v_{g|s}$ as a deviation payoff since it describes the expected payoff from adopting a gambling strategy when sentiments and other banks' strategies are consistent with a safe equilibrium.

Similarly, the gambling equilibrium exists under the equilibrium condition

$$v_g \geq v_{s|g} \tag{1.44}$$

Figure 1.6: **Example with multiple equilibria**



where v_g is the expected payoff under the gambling equilibrium given by (1.32) and $v_{s|g}$ is the expected payoff from a 'deviation to the safe strategy'. I elaborate further on these deviations below.

There are three possible equilibrium outcomes. When (1.43) is satisfied and (1.44) is not, banks follow a safe strategy regardless of household sentiments and there is a unique safe equilibrium. In this case, bad sentiments are not self-confirming and thus may not exist. In contrast, when (1.44) is satisfied and (1.43) is violated, there is a unique gambling equilibrium and only bad sentiments exist. Finally, when both conditions are satisfied, banks follow a safe strategy under good sentiments and gamble under bad sentiments such that there are multiple equilibria.

I use Figure 1.6 as an informal example to provide further intuition about the mechanism behind multiple equilibria. In the interest of a clear exposition, I focus on a case where the solvency constraint remains slack regardless of household sentiments.[22] Under good sentiments, the representative bank faces the deposit demand schedule depicted by the dotted line, where the deposit threshold $\bar{d}(\gamma_s)$ is consistent with a safe strategy. This permits the bank to raise sufficient deposits to satisfy its optimality condition for lending (1.35) without reducing the price of its deposits below the risk-free level q^* under a safe strategy. It then finds it optimal to adopt a safe strategy such that there is a safe equilibrium E_s and good sentiments are confirmed.

[22] This mechanism becomes even stronger when the solvency constraint binds, since the downward pivot in the deposit demand schedule under bad sentiments leads to a tightening of the solvency constraint as shown in the third panel of Figure 1.7.

When there is a shift to bad sentiments, the expectation of a high aggregate risk exposure $\gamma_g > \gamma_s$ leads to an inward shift of the deposit threshold to $\bar{d}\left(\gamma_g\right) < \bar{d}\left(\gamma_s\right)$. The deposit demand schedule then pivots downward in the non-verifiable region $d \in (\bar{d}\left(\gamma_g\right), \bar{d}\left(\gamma_s\right)]$. Because of this deterioration in the bank's borrowing conditions, the quantity and price of deposits fall to $E_{s|g}$ under the safe strategy. This leads to a decline in the expected payoff associated with this strategy. If the bank finds it optimal to deviate to a gambling strategy that leads to the outcome E_g, bad sentiments are also confirmed and there are multiple equilibria.

Below, I briefly describe the deviations to gambling and safe strategies before characterizing the parameter boundaries for the three equilibrium regions (with a unique safe equilibrium, a unique gambling equilibrium, and multiplicity) in Section 1.3.4.

Deviation to the gambling strategy Consider a deviation to the gambling strategy when sentiments and other banks' strategies correspond to the safe equilibrium in Section 1.3.1. Under such a deviation, the bank's strategy is guided by the first order conditions (1.20) and (1.21), yielding valuations for deposits and loans that are consistent with a gambling equilibrium.

However, the quantity of loans purchased by the deviating bank

$$l_{g|s} = \left(q_g^l\right)^{\frac{\alpha}{1-\alpha}} (\alpha A)^{\frac{1}{1-\alpha}} - \frac{1-\phi}{\phi} l_s \tag{1.45}$$

differs from its gambling equilibrium counterpart, which is given by (1.31). This is because the remaining banks each purchase an amount l_s consistent with the safe equilibrium, thus driving up loan prices. The negative relationship between $l_{g|s}$ and l_s follows directly from the upward-sloping loan supply schedule. As other banks provide more loans, the scope for lending by the deviating bank diminishes. This also reduces the expected payoff from deviation which is increasing in bank lending as in the gambling equilibrium

$$v_{g|s} = (1-P)\,\mu_l l_{g|s} + \frac{n}{q^*} \tag{1.46}$$

Lemma 1.4 builds upon this intuition to show that the safe equilibrium is always satisfied when the solvency constraint is slack.

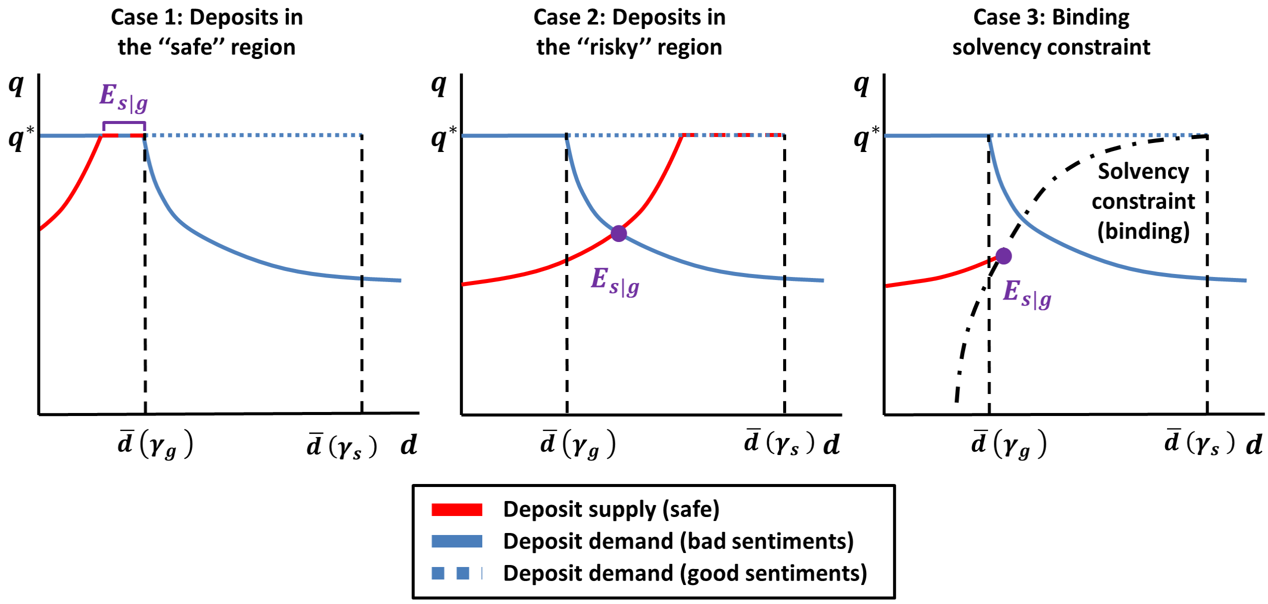
Lemma 1.4 *The parameter restrictions given by (1.15) are sufficient to show that*

$$v_s > v_{g|s} \ \forall \ n \geq n_c$$

Proof. Provided in Appendix A3.4. ■

Recall from Lemma 1.3 that l_s is increasing in net worth n when the solvency constraint

Figure 1.7: **Deviation to the safe Strategy**



Note: Deposit demand is attained by combining (1.14) with (1.38) under good sentiments and (1.29) under bad sentiments. The deposit supply curve stems from the combination of (1.3), (1.25), (1.26) and (1.47). The solvency constraint is given by (1.48).

binds. It is thus possible for (1.43) to be violated at a level of net worth below n_c such that there is a unique gambling equilibrium. I elaborate further on this in Section 1.3.4 after describing deviations to the safe strategy.

Deviation to the safe strategy Under a deviation to the safe strategy, the bank follows the first order conditions (1.24)-(1.26) but faces a deposit demand schedule

$$q\left(\gamma_g, d\right) = \begin{cases} q^* & \text{for } d \leq \bar{d}\left(\gamma_g\right) \\ q^b + \frac{P\theta^b}{1-P}\frac{n}{d} & \text{for } d > \bar{d}\left(\gamma_g\right) \end{cases} \tag{1.47}$$

$$\bar{d}\left(\gamma_g\right) = \frac{\theta^b}{q^b - \theta^b q^*} n$$

consistent with bad sentiments. As the bank's actual aggregate risk exposure diverges from household expectations, the solvency constraint no longer corresponds to the deposit threshold $\bar{d}\left(\gamma_g\right)$. This opens up the possibility that the bank may move to the risky region of the deposit demand schedule despite satisfying the solvency constraint.

There are thus three possible cases of the deviation to the safe strategy which are valid at different regions of bank net worth n. In the interest of brevity, I relegate the characterization of these cases to Appendix A2 and instead provide a brief description of each case with the aid of Figure 1.7. In the first case, the deviating bank has a slack solvency constraint and remains in the safe region of the deposit threshold $d_{s|g} \leq \bar{d}\left(\gamma_g\right)$. This case is nearly identical to Case 1 of the safe equilibrium, except for a rise in the boundary level of net worth required for this case to be valid to $n_{r|g} > n_c$ due to the inwards shift of the deposit threshold under bad sentiments.[23]

In the second case, the shift to bad sentiments leaves the optimal level of deposits in the "risky" region of the deposit demand schedule, while the actual solvency constraint remains slack. The decline in the value of deposits to $q_{s|g} < q^*$ leads to a fall in bank lending and expected payoff. Finally, in the third case, the solvency constraint binds, creating a wedge between deposit demand and deposit supply and further reducing lending and expected payoff. Note that the solvency constraint, which is given by

$$\left(q^l_{s|g} - q\left(\gamma_g, d_{s|g}\right)\theta^l\right) l_{s|g} = n \tag{1.48}$$

tightens in response to a decline in the price of deposits.

1.3.4 Regions of equilibria

There are three possible equilibrium outcomes to the model. First, there is a unique gambling equilibrium when banks follow a gambling strategy regardless of household sentiments. Second, there are multiple equilibria if banks adopt a safe strategy under good sentiments and a gambling strategy under bad sentiments such that both good and bad sentiments are self-fulfilling. Third, there is a unique safe equilibrium when banks follow a safe strategy regardless of household sentiments. I denote the regions of parameters where these outcomes are prevalent as $\mathcal{G}$, $\mathcal{M}$ and $\mathcal{S}$ respectively.

Proposition 1.2 expresses the equilibrium conditions (1.43), (1.44) as parameter boundaries for these regions.

Proposition 1.2 *Under the parameter restrictions given by (1.15), the mapping of equilibrium regions across net worth n is given by*

$$\mathcal{E}(n) = \begin{cases} \mathcal{G} & \textit{if } n \leq \underline{n} \\ \mathcal{M} & \textit{if } \underline{n} < n < \bar{n} \\ \mathcal{S} & \textit{if } n > \underline{n} \end{cases} \tag{1.49}$$

[23] See Appendix A2 for a definition for $n_{r|g}$.

where $\underline{n} < n_c$ is implicity defined by the expression

$$\begin{aligned}\underline{n} &= \left(\frac{1}{\phi}\right)^{\frac{1-\alpha}{\alpha}} \left(\frac{1}{A\alpha}\frac{q^*(1-P)\mu_l\left(q_g^l\right)^{\frac{\alpha}{1-\alpha}}(A\alpha)^{\frac{1}{1-\alpha}}+\underline{n}}{q^*(1-P)\left[1-\theta^l+\mu_l\frac{1-\phi}{\phi}\right]}\right)^{\frac{1}{\alpha}} \\ &\quad -\theta^l\frac{q^*(1-P)\mu_l\left(q_g^l\right)^{\frac{\alpha}{1-\alpha}}(A\alpha)^{\frac{1}{1-\alpha}}+\underline{n}}{(1-P)\left[\left(1-\theta^l\right)+\mu_l\frac{1-\phi}{\phi}\right]}\end{aligned} \tag{1.50}$$

and $\bar{n}$ is given by

$$\bar{n} \equiv \frac{(1-P)\,q^*}{P}\left[(1-P)+P\theta^l(1-\phi)-\left(1-P+P\theta^l\right)^{\frac{1}{1-\alpha}}\right]\left((1-\mu_l)\,q^b\right)^{\frac{\alpha}{1-\alpha}}(\alpha A)^{\frac{1}{1-\alpha}}\mu_l \tag{1.51}$$

under the sufficient conditions $\alpha \in (0, \frac{1}{2}]$, $\phi \in (0, \frac{1}{2}]$.

Proof. Provided in Appendix A3.6. ∎

Note that (1.49) indicates a monotonic ordering of equilibria across bank net worth n. Since $\underline{n} < n_c$, there is no overlap between $\mathcal{M}$ and the case of the safe equilibrium with a slack solvency constraint. Without an upper bound to bank net worth n, this is sufficient to show that $\mathcal{S}$ is non-empty. Proposition 1.3 describes the conditions under which $\{\mathcal{G}, \mathcal{M}\}$ are also non-empty.

Proposition 1.3 *Under the parameter restrictions given by (1.15), the non-emptiness of regions $\{\mathcal{G}, \mathcal{M}\}$ depends on where θ^l stands with respect to the boundary $\underline{\theta}^l$, which is implicitly defined by the expression*

$$(1-\phi)+\phi\frac{1-\underline{\theta}^l}{\mu_l} = \left(\frac{(1-\mu_l)\left(1-P+P\theta^b\right)}{\underline{\theta}^l}\right)^{\frac{\alpha}{1-\alpha}} \tag{1.52}$$

There are two possible cases.

Case 1 *If $\theta^l \geq \underline{\theta}^l$, $\mathcal{G}$ is empty and $\mathcal{M}$ is always non-empty.*

Case 2 *If $\theta^l < \underline{\theta}^l$, $\mathcal{G}$ is non-empty and a sufficient condition for $\mathcal{M}$ to be non-empty is*

$$\frac{\theta^b}{\alpha+(1-\alpha)\,\phi} > 1-P+P\theta^b \tag{1.53}$$

Proof. Provided in Appendix A3.7. ∎

1.4 Policy analysis

This section evaluates policy interventions aimed at strengthening the banking sector and reinvigorating bank lending. It is clear from Section 1.3.4 that both of these aims can be achieved with a capital injection to the banking sector that directly increases bank net worth n. However, this requires a significant transfer of resources at a time of uncertainty about future economic fundamentals.

Instead, I focus on unconventional interventions that can be implemented by the central bank and macroprudential policy measures. Section 1.4.1 considers (non-targeted) liquidity provision to the banking sector by the central bank. Section 1.4.2 proposes an alternative measure, *targeted* liquidity provision, where the central bank provides liquidity conditional on bank leverage. Finally, Section 1.4.3 shows that the findings from Sections 1.4.1 and 1.4.2 can be generalized to provide insights for deposit insurance and a range of macroprudential policies.

1.4.1 Liquidity provision

I incorporate liquidity provision into the model by allowing each bank to issue debt $d^c \leq \bar{d}^c$ to the central bank at a risk-free price q^*.[24] With access to central bank liquidity, the representative bank's budget constraint and profits become

$$\begin{aligned} n + qd + q^* d^c &= q^b b + q^l l \\ \pi &= \max\{0, l + b - d - d^c\} \\ \underline{\pi} &= \max\left\{0, \theta^l l + \theta^b b - d - d^c\right\} \end{aligned}$$

Crucially, the effects of central bank liquidity hinge on whether it leads to a transfer of bank insolvency risk from depositors to the central bank.

Liquidity provision with no risk transfer Consider first the case with no risk transfer such that liabilities to the central bank have greater seniority than deposits. In other words, debt repayments to the central bank take priority over deposits in the event that the bank becomes insolvent. This ensures that the central bank is not exposed to any potential losses at the expense of diluting depositors' claim to bank revenues.[25]

[24] I abstain from collateral requirements on debt issued to the central bank. In practice, collateral requirements do not preclude the form of gambling considered here as long as risky bonds are eligible as collateral. Placing a haircut on risky assets pledged as collateral is equivalent to a reduction in $\bar{d}_c$, and the intervention becomes completely ineffective when risky assets are ineligible as a collateral (i.e. a haircut of 100% leads to $\bar{d}_c = 0$).

[25] This is true unless the liquidity provided by the central bank exceeds total bank revenues under weak fundamentals. The restriction $\bar{d}^c \leq \frac{\theta^b}{1-\theta^b} d$ is sufficient to preclude this and satisfied under plausible values for $\bar{d}^c$

The dilution of deposits proves to be crucial in undermining the policy intervention. It creates a negative relationship between the amount of central bank liquidity d^c held by the bank and the recovery rate of deposits θ. This is reflected in the deposit demand schedule, which is now given by

$$q^c(\tilde{\gamma}, d, d^c) = \begin{cases} q^* & \text{for } d + d^c \leq \bar{d}(\tilde{\gamma}) \\ q^* \frac{1 - P + P\left(\left(\tilde{\gamma}\frac{\theta^b}{q^b} + (1-\tilde{\gamma})\frac{\theta^l}{q^l}\right)\left(\frac{n + q^* d^c}{d}\right) - \frac{d^c}{d}\right)}{1 - q^* P\left(\tilde{\gamma}\frac{\theta^b}{q^b} + (1-\tilde{\gamma})\frac{\theta^l}{q^l}\right)} & \text{for } d + d^c > \bar{d}(\tilde{\gamma}) \end{cases} \tag{1.54}$$

where the deposit threshold $\bar{d}(\tilde{\gamma})$ remains unchanged. When the parameter restrictions in (1.15) are satisfied, a rise in central bank liquidity d^c leads to an inward shift in the deposit demand schedule. Using (1.14) and (1.54), it is easy to show that the bank's ability to raise funds is independent of d^c such that

$$q^c(\tilde{\gamma}, d, d^c)\, d + d^c = q(\tilde{\gamma}, d)\, d \ \ \forall\ d^c \leq \bar{d}^c$$

where $q(\tilde{\gamma}, d)$ is the deposit demand schedule in the absence of liquidity provision. This indicates that the deterioration in bank borrowing conditions due to dilution exactly offsets the gains from central bank liquidity. Consequently, liquidity provision is completely ineffective without a risk transfer to the central bank.

Liquidity provision with risk transfer Now consider the case where the repayment of deposits takes priority over obligations to the central bank. This constitutes an implicit transfer of bank insolvency risk from depositors to the central bank as the recovery rate of deposits increases at the expense of central bank losses. The deposit demand schedule is then given by the expressions

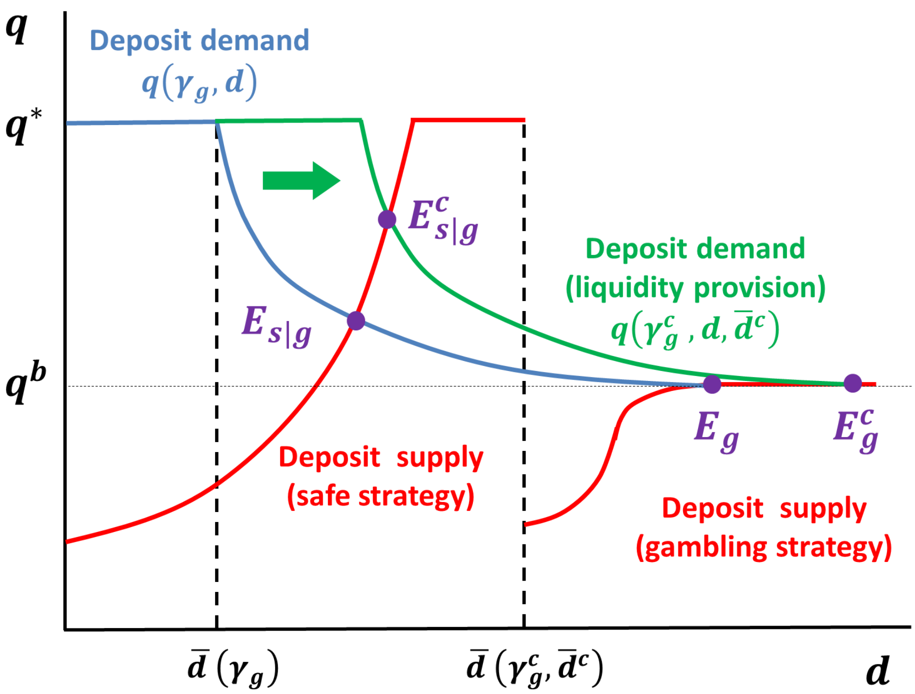
$$q^c(\tilde{\gamma}, d, d^c) = \left\{ \begin{array}{ll} q^* & \text{for } d \leq \bar{d}^c(\tilde{\gamma}, d^c) \\ q^* \frac{1 - P + P\left(\tilde{\gamma}\frac{\theta^b}{q^b} + (1-\tilde{\gamma})\frac{\theta^l}{q^l}\right)\left(\frac{n + q^* d^c}{d}\right)}{1 - q^* P\left(\tilde{\gamma}\frac{\theta^b}{q^b} + (1-\tilde{\gamma})\frac{\theta^l}{q^l}\right)} & \text{for } d > \bar{d}^c(\tilde{\gamma}, d^c) \end{array} \right\}, \tag{1.55}$$

$$\bar{d}^c(\tilde{\gamma}, d^c) = \frac{\left(\tilde{\gamma}\frac{\theta^b}{q^b} + (1-\tilde{\gamma})\frac{\theta^l}{q^l}\right)(n + q^* d^c)}{1 - q^*\left(\tilde{\gamma}\frac{\theta^b}{q^b} + (1-\tilde{\gamma})\frac{\theta^l}{q^l}\right)}$$

Rather than being diluted, the expected value of deposits increases in central bank liquidity d^c, causing an outwards shift in the deposit demand schedule as shown in Figure 1.8.

Note that the first order conditions (1.20) and (1.21) for the gambling strategy remain unchanged. Therefore, the bank does not change its lending to firms in response to liquidity provision. Instead, it takes advantage of the outward shift in its deposit demand schedule to

Figure 1.8: **Liquidity provision with risk transfer**



Note: The deposit demand schedule is attained by combining (1.55) with (1.29). The deposit supply curve stems from the combination of (1.3), (1.25), (1.26) and (1.47). Only the case with a slack solvency constraint is included. For the remaining cases see Figure 1.7.

increase its deposits and risky asset purchases until its borrowing costs return to their level prior to the intervention. Therefore, the recovery rate of deposits θ also remains at its pre-intervention level such that depositors face the same amount of insolvency risk. In other words, the risk transfer simply provides the bank with an opportunity to increase the extent of its gamble on aggregate risk at the expense of the central bank. Accordingly, the expected payoff associated with a deviation to gambling increases.

Proposition 1.4 indicates that this intervention backfires by eliminating the safe equilibrium.

Proposition 1.4 *The gambling equilibrium is unique for all n when*

$$\bar{d}^c > \tilde{d}^c \equiv \frac{\mu_l \left(\alpha A \left(q^l_g\right)^\alpha\right)^{\frac{1}{1-\alpha}}}{P} \left[\left(1 - P + \phi P \theta^l\right) \left(\frac{1 - P + P\theta^l}{1 - P + P\theta^b}\right)^{\frac{\alpha}{1-\alpha}} - (1 - P)\right]$$

When $\bar{d}^c \leq \tilde{d}^c$, the gambling equilibrium is unique for $n \leq \underline{n}$ where $\underline{n}$ is implicity defined by the *expression*

$$\underline{n} = \left(\frac{1}{\phi}\right)^{\frac{1-\alpha}{\alpha}} \left(\frac{1}{A\alpha}\frac{q^*\left(1-P\right)\mu_l\left(q_g^l\right)^{\frac{\alpha}{1-\alpha}}\left(A\alpha\right)^{\frac{1}{1-\alpha}}+\underline{n}+q^*\bar{d}^c}{q^*\left(1-P\right)\left[1-\theta^l+\mu_l\frac{1-\phi}{\phi}\right]}\right)^{\frac{1}{\alpha}}$$
$$-\theta^l\frac{q^*\left(1-P\right)\mu_l\left(q_g^l\right)^{\frac{\alpha}{1-\alpha}}\left(A\alpha\right)^{\frac{1}{1-\alpha}}+\underline{n}+q^*\bar{d}^c}{\left(1-P\right)\left[\left(1-\theta^l\right)+\mu_l\frac{1-\phi}{\phi}\right]}$$

and

$$\frac{\partial\underline{n}}{\partial\bar{d}^c}>0$$

Proof. Provided in Appendix A3.8. ■

The first part of the proposition shows that when banks have access to central bank liquidity in excess of an upper bound $\tilde{d}^c$, they find it optimal to gamble even when the solvency constraint is slack. Gambling then becomes the unique equilibrium regardless of bank net worth. The second part shows that even for $\bar{d}^c \leq \tilde{d}^c$, the intervention shifts up the boundary of net worth $\underline{n}$ below which there is a unique gambling equilibrium.

This negative result stems from the inability of non-targeted interventions to distinguish between banking strategies, which in turn leads to a trade-off between alleviating funding conditions under the safe strategy and strengthening incentives to gamble. In the next section, I propose a targeted intervention that overcomes this trade-off.

1.4.2 Targeted liquidity provision

Under targeted liquidity provision, the central bank offers a liquidity schedule $\bar{d}^c\left(d,n\right)$ conditional on deposits and bank net worth. By offering a liquidity schedule

$$\bar{d}^c\left(n,d\right)=\frac{\left(\frac{\theta^l}{q_s^l}-\frac{\theta^b}{q^b}\right)q_s^l l_s+\frac{\theta^b}{q^b}n}{1-\frac{\theta^b}{q^b}q^*}-d \tag{1.56}$$

which overlaps with the solvency constraint under good sentiments, the central bank can completely insulate the banking sector from shifts in depositor sentiments.

By design, the schedule has no impact on banks' funding conditions under good sentiments. When there is a shift to bad sentiments, however, it provides banks with low cost liquidity in a manner that artificially re-creates the funding conditions under good sentiments. It then follows directly from the equilibrium conditions (1.43), (1.44) that bad sentiments cease to be self-fulfilling throughout the multiplicity region. The intervention remains strictly off-equilibrium when it is successful, since banks are indifferent between central bank and deposit funding in the safe equilibrium.

The conditionalities on (n, d) are crucial for the success of the intervention. By placing an upper bound on participating banks' leverage, these conditionalities ensure that banks do not find it optimal to take up central bank liquidity under the gambling strategy. This overcomes the trade-off faced by non-targeted liquidity provision, allowing the intervention to improve banks' funding conditions under the safe strategy without increasing incentives to gamble.

Note that the results from Section 1.4.1 with regard to the irrelevance of liquidity provision without a risk transfer remain valid. Therefore, at least in principle, the targeted intervention requires that the central bank becomes exposed to bank insolvency risk.[26] In practice, however, the central bank never faces losses under targeted liquidity provision. This is not just due to the fact that successful interventions are never implemented in equilibrium. Even if the liquidity schedule is offered in the region with a unique gambling equilibrium such that the intervention fails, the conditionalities ensure that banks do not take up central bank liquidity in a gambling equilibrium.

The role of the conditionalities is thus twofold. First, they drive a wedge between the safe and gambling strategies and allow the central bank to make the former more attractive, thereby eliminating multiplicity in favour of the safe equilibrium. Second, they ensure that the central bank is not subject to losses even when the intervention is unsuccessful.

Finally, note that the central bank does not need to observe the aggregate risk exposure γ in order to implement this intervention. This raises the question as to why the central bank is capable of carrying out this intervention while the households cannot. The answer lies in the ability of the central bank to internalize the equilibrium-switching effects of its behaviour, and thus commit to the liquidity schedule in (1.56). In contrast, for atomistic households that take sentiments as given, (1.56) is strictly sub-optimal to the deposit demand schedule. In other words, targeted liquidity provision resolves a coordination problem between banks and depositors.

1.4.3 Deposit insurance and macroprudential regulation

In this section, I generalize the findings from Sections 1.4.1 and 1.4.2 to a wider set of policy instruments. To begin with, consider deposit insurance in the form of a limited amount of funds F/ϕ dedicated to increasing the recovery rate θ of deposits, which can then be written as

$$\theta = \min\left\{1, \left(\tilde{\gamma}\frac{\theta^b}{q^b} + (1-\tilde{\gamma})\frac{\theta^l}{q^l}\right)\left(\frac{n}{d} + q\right) + \frac{F}{d}\right\}$$

[26] This does not necessarily need to take the form of an explicit arrangement where depositors have greater seniority. When the central bank has priority in debt repayments, providing the liquidity schedule above under bad sentiments completely crowds out deposit funding. Without deposits to act as a buffer, bank insolvency results in losses for the central bank.

This leads to the following deposit demand schedule

$$q^F(\tilde{\gamma}, d, F) = \begin{cases} q^* & \text{for } d \le \bar{d}^F(\tilde{\gamma}, F) \\ q^* \frac{1-P+P\left(\left(\tilde{\gamma}\frac{\theta^b}{q^b}+(1-\tilde{\gamma})\frac{\theta^l}{q^l}\right)n+F\right)\frac{1}{d}}{1-q^*P\left(\tilde{\gamma}\frac{\theta^b}{q^b}+(1-\tilde{\gamma})\frac{\theta^l}{q^l}\right)} & \text{for } d > \bar{d}^F(\tilde{\gamma}, F) \end{cases} \tag{1.57}$$

$$\bar{d}^F(\tilde{\gamma}, F) = \frac{\left(\tilde{\gamma}\frac{\theta^b}{q^b} + (1-\tilde{\gamma})\frac{\theta^l}{q^l}\right)n + F}{1 - q^*\left(\tilde{\gamma}\frac{\theta^b}{q^b} + (1-\tilde{\gamma})\frac{\theta^l}{q^l}\right)}$$

which indicates that deposit insurance leads to an outward shift deposit demand. Proposition 1.5 shows that, on its own, deposit insurance backfires in the same manner as non-targeted liquidity provision (with risk transfer).

Proposition 1.5 *For any arbitrary $\varepsilon \ge 0$*

$$q^F(\tilde{\gamma}, d, \varepsilon)\, d = q^c(\tilde{\gamma}, d, \varepsilon)\, d + q^*\varepsilon$$

Proof. Provided in Appendix A3.9. ■

As before, the negative result stems from the trade-off between alleviating funding conditions and strengthening incentives to gamble. This trade-off can be overcome with the use of macroprudential regulation. Specifically, the combination of deposit insurance with a regulatory constraint on bank liabilities can lead to a similar outcome to targeted liquidity provision. This is achieved by dedicating sufficient funds to deposit insurance to offset the effects of a shift to bad sentiments on the deposit demand schedule

$$F = \left(\frac{\theta^l}{q^l_s} - \frac{\theta^b}{q^b}\right) q^l_s l_s$$

and imposing a regulatory constraint that overlaps with the solvency constraint in the safe equilibrium[27]

$$d \le \frac{F + \frac{\theta^b}{q^b} n}{1 - \frac{\theta^b}{q^b} q^*}$$

Finally, note that the same outcome can be achieved with alternative forms of macroprudential regulation. For example, the liability constraint above is interchangeable with a constraint on asset holdings or capital requirements in a richer environment with equity issuance, provided that there is a positive risk-weight attached to aggregate risky assets.

[27] If participation in the deposit insurance and macroprudential regulation scheme is non-voluntary, the failure of the policy may lead to the use of deposit insurance funds in equilibrium. In the region with a unique gambling equilibrium, banks respond to a non-voluntary scheme by following a gambling strategy despite satisfying the regulatory constraint.

1.5 Conclusion

This chapter proposes a general equilibrium model with optimizing banks and depositors to analyse economic vulnerability financial and debt crises, and draw insights for policy design. An important finding emerges as a consequence: Opacity in bank balance sheets leads to strategic complementarities between banks and depositors as depositors demand a return on deposits according to their expectations on bank risk-taking, and banks determine their risk-taking strategies according to their funding costs. This raises the possibility of multiple equilibria, where a safe equilibrium is characterized by low risk-taking and funding costs, and a gambling equilibrium is associated with bank insolvency risk and high funding costs.

The model also provides a framework for policy analysis. As a novel insight, it indicates that a key prerequisite for successful liquidity interventions by central banks is that they provide some risk-sharing with depositors. Otherwise, liquidity interventions are completely ineffective as depositors raise bank funding costs in anticipation of the dilution of their claims to bank revenues. A second insight pertains to the targeting of interventions. Non-targeted interventions that provide liquidity (and risk sharing) unconditionally may eliminate the safe equilibrium. This is because they face a trade-off between alleviating funding constraints and strengthening incentives to gamble. It is possible to overcome this trade-off with a targeted intervention that provides liquidity conditional on bank leverage.

Finally, note that the mechanisms considered in this chapter are relevant under two conditions: First, there must be aggregate risk without sufficiently strict regulation to prevent banks from becoming exposed to it. Second, government guarantees on the banking sector must be incomplete or incompletely credible. These conditions are often satisfied in times of financial turmoil, episodes of sovereign default, and when currency pegs come under pressure.

Chapter 2

Gambling Traps

Abstract

I propose a dynamic general equilibrium model where strategic interactions between banks and depositors may lead to "gambling traps" associated with endogenous bank fragility and slow recovery from crises. In a gambling trap, depositor expectations of high bank risk-taking become self-fulfilling and high bank funding costs hinder the accumulation of bank net worth, leading to a persistent drop in investment and output. I bring the model to bear on the European sovereign debt crisis, in the course of which under-capitalized banks in default-risky countries experienced an increase in funding costs and raised their holdings of domestic government debt. The model is quantified using Portuguese data and accounts for macroeconomic dynamics in Portugal in 2010-2016. Subsidized loans to banks, similar to the ECB's longer-term refinancing operations (LTRO), strengthen incentives to gamble and may perpetuate gambling traps.

Keywords: Bank risk-taking; Financial intermediation; Sovereign debt crisis
JEL codes: E44, E58, F34, G01, G21, H63

2.1 Introduction

Recent financial and debt crises are characterized by sluggish recoveries and fragility in the banking sector leading to high bank funding costs. This challenges current theoretical models that typically abstract from bank funding costs, assuming that banks have access to deposits at the risk-free rate (see e.g. Brunnermeier and Sannikov, 2014; Gertler and Karadi, 2011; Gertler and Kiyotaki, 2010).

In this chapter, I incorporate the framework proposed in Chapter 1 into a dynamic general equilibrium model in order to analyze macroeconomic dynamics and vulnerability to crises in the light of strategic interactions between banks and depositors. I build upon the insight that depositor expectations on bank risk-taking may become self-fulfilling in economies with high aggregate risk and an under-capitalized banking sector. Most importantly, I show that these economies are vulnerable to "gambling traps" characterized by a prolonged period of financial fragility and endogenously slow recovery from crises.

I bring this theoretical model to bear on the European sovereign debt crisis, its transmission to economic activity and policy debates on interventions in support of the banking sector. In doing so, I bring forward two key empirical facts to motivate my model. In countries hit by the sovereign debt crisis, under-capitalized banks increased their exposure to sovereign risk by investing heavily in their own government's debt. In these countries, there is also significant co-movement between yield spreads on sovereign bonds and deposit interest rates.

I develop my analysis by specifying a dynamic small open economy model with households, firms, and a banking sector. Banks collect deposits from households and choose their portfolios of sovereign bonds and loans to firms; households lend to banks on terms that depend on bank solvency prospects; firms invest. The government issues default-risky bonds.

As in the first chapter, strategic complementarities between banks and depositors lead to the possibility of multiple equilibria. When bank net worth is high and/or market sentiments are "good", there is a "safe equilibrium" where banks keep their exposure to government debt low and depositors accept low interest rates. With low bank net worth and "bad" market sentiments, a "gambling equilibrium" emerges. In this equilibrium, depositors expect banks to have a high exposure to risky government debt and require a risk premium. In response to high funding costs, banks find it optimal to gamble on risky sovereign debt at the expense of credit to firms.

The key intuition in this chapter is that high funding costs in the gambling equilibrium hinder the accumulation of bank net worth. This has important consequences for macroeconomic adjustment to shocks, since the economy may become stuck in a gambling trap with stagnating bank net worth and repeated realizations of the gambling equilibrium. In a gambling trap, banks respond to sovereign risk shocks by increasing their exposure to domestic government debt rather than deleveraging, leading to a persistent drop in output and financial fragility. Persistence here is endogenous, and absent in the safe equilibrium, where banks deleverage and all the adjustment (in credit and output) is front-loaded and short-lived.

I bring this model to data by calibrating it to Portugal over 2010-2016 and simulating it under a series of sovereign risk shocks that emulate Portuguese sovereign bond yields. The simulation indicates that the Portuguese economy is vulnerable to multiple equilibria, and shows that a sequence of bad sentiments (i.e. a gambling trap) can account for dynamics of key macroeconomic and financial variables during the sovereign debt crisis.

The model also provides novel insights for policy design. In a policy experiment, I consider an intervention similar to the ECB's longer-term refinancing operations (LTRO) where the central bank temporarily (and unconditionally) provides cheap liquidity to the banking sector. I show that this intervention strengthens incentives to gamble such that the economy remains stuck in a gambling trap for its duration.

Relationship to the literature This chapter is related to the literature on macroeconomic dynamics under financial frictions. Specifically, it relates to a strand of literature that analyzes financial and macroeconomic adjustment to shocks when banks are balance sheet constrained. In this literature, banks channel funds from households to productive investment opportunities and face an occassionally binding constraint on their leverage.[28] For example, in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011), the balance sheet constraint arises as a commitment mechanism to prevent bank managers from diverting funds to themselves, while in Meh and Moran (2010) it incentivizes banks to monitor their borrowers.

When the balance sheet constraint tightens and/or bank net worth declines, banks are forced to deleverage and reduce intermediation. This leads to a decline in investment and output, but also creates excess returns in intermediation that re-build bank net worth, paving the way to a recovery. He and Krishnamurthy (2014) focus on anticipation effects on bank behaviour associated with the possibility that the constraint may become binding in the future. Brunnermeier and Sannikov (2014) consider a similar transmission mechanism in a highly non-linear environment with fire-sales.

A common feature of studies in this strand of literature is that the leverage constraint rules out banking default in equilibrium, thereby ensuring that bank have access to funds at the risk-free rate. In other words, all of the financial adjustment is pushed to the quantity of intermediation rather than costs thereof (i.e. funding costs). This stands in contrast to evidence from recent crises such as the financial crisis of 2007-09 and the European debt crisis, where bank funding costs increased in response to a perceived rise in bank fragility.

The main contribution of this chapter is to uncover an alternative mechanism of adjustment

[28] See Brunnermeier et al. (2013) for a comprehensive survey of this literature.

to crises. When banks pursue a "safe strategy", they satisfy an occassionally binding solvency constraint and macroeconomic adjustment to crises is similar to the above literature. The solvency constraint itself is similar to the balance sheet constraint described above, but arises solely due to imperfect transparency in banks' portfolios. Banks may also choose to pursue a "gambling strategy" which entails forming a risky portfolio and may lead to default in equilibrium. Under this strategy, banks respond to adverse shocks by increasing their risk-taking and bank funding costs rise in response. I show that both of these strategies may arise in equilibrium, and the type of equilibrium depends on bank net worth and potentially self-fulfilling market sentiments about bank risk-taking.

Analysing macroeconomic dynamics under these equilibria yields two important insights. First, there are multiple adjustment paths since the prevalent equilibrium at a given time period determines future net worth and hence the possible equilibrium types in future periods. Most importantly, I show that net worth recovers slowly in the gambling equilibrium such that the decline in output following a negative shock becomes endogenously persistent. Second, policy interventions have equilibrium-switching effects which are significant precisely because of the considerable differences in adjustment paths to crises under the two equilibria.

This chapter is also related to the literature on banking and sovereign debt. The strong positive relationship between sovereign risk and private borrowing costs is well documented (see e.g. Acharya et al., 2014b; Popov and Van Horen, 2015) and incorporated in reduced form by several studies including Neumeyer and Perri (2005), Uribe and Yue (2006), and Corsetti et al. (2013). This chapter builds upon a growing literature that provides microfoundations for the aforementioned relationship by exploring the links between sovereign default and the domestic banking sector. The existing literature can be divided into two main strands according to the channel of transmission.

In the first strand, which includes Basu (2010), Gennaioli et al. (2014), Sosa Padilla (2015), and Perez (2015), agency frictions constrain banks' ability to leverage.[29] Sovereign default tightens this constraint by weakening bank balance sheets, forcing banks to deleverage and reduce financial intermediation. Bolton and Jeanne (2011) and Bocola (2016) show that the ex-ante anticipation of sovereign default is sufficient to generate these effects, leading to a decline in sovereign bond purchases as well as intermediation. Like Bocola (2016), I treat sovereign default risk as driven by some exogenous latent factor. Abstracting from the government's default decision allows me to focus sharply on the properties of the novel mechanism my model is about.

In the second strand, depositors in domestic banks are shielded from potential losses in the

[29] Sosa Padilla (2015) depicts bank liabilities as a constant flow. This leads to a similar transmission mechanism as the studies with agency frictions since a haircut on sovereign bonds directly reduces the funds available for intermediation. Brutti (2011) and Perez (2015) consider the effects of sovereign default on banks' ability to store liquidity. Sandleris (2014) considers the signalling effects of sovereign default.

event of sovereign default due to a variety of reasons, such as a bailout of the banking sector in Brunnermeier et al. (2016) and Farhi and Tirole (2017), deposit insurance in Livshits and Schoors (2009) and selective sovereign default in Broner et al. (2014). This undermines market discipline such that banks respond to a rise in sovereign risk by increasing their domestic debt purchases in order to take advantage of high yields.

By replacing the respective assumptions in these strands with limited liability, opacity of bank balance sheets and balance sheet costs of domestic sovereign default, I develop a framework with two possible equilibrium outcomes; a safe equilibrium which gives rise to a transmission mechanism similar to the first strand, and a gambling equilibrium that resembles the second strand. A closely related study is that of Acharya et al. (2014a). They also consider a framework where banks face insolvency risk, but focus on the government's bailout decision rather than strategic interactions between banks and depositors.

This chapter is also related to a recent strand of research that considers the relationship between sovereign risk and banking fragility. Cooper and Nikolov (2013) analyse the interaction between self-fulfilling debt crises as in Calvo (1988) and Diamond and Dybvig (1983) bank runs, whereas Leonello (2016) considers similar interactions in a global games framework. Two layers of strategic complementarities are overlaid in these studies; one across sovereign debt-holders and another across depositors. This chapter instead focuses on strategic complementarities *between* the optimal responses of banks and depositors.

This chapter also draws from a rich literature on the repatriation of sovereign debt in open economies. Merler and Pisani-Ferry (2012) document this in the context of the European sovereign debt crisis.[30] Three alternative hypotheses have come to the fore as a potential explanation. First, creditor discrimination theories suggest that sovereign risk drives a wedge between the valuation of sovereign debt by domestic and foreign agents due to anticipated discrimination in favour of the former during a default event (Broner et al., 2014). Second, moral suasion theories suggest that governments in need of funding incentivize or directly coerce domestic banks to purchase their debt (Chari et al., 2016). The third hypothesis corresponds to the gambling mechanism considered here: under-capitalized banks find default-risky domestic sovereign debt attractive for risk-shifting purposes, since its payoff is positively correlated with their solvency prospects.

Brutti and Sauré (2016) find evidence in favour of creditor discrimination, while Acharya and Steffen (2015), Battistini et al. (2014) and Altavilla et al. (2016) lend support to both moral suasion and gambling hypotheses. De Marco and Macchiavelli (2016), Becker and Ivashina (2014) and Ongena et al. (2016) provide additional evidence for moral suasion. Acharya and Steffen (2015) find evidence for gambling by showing that banks with high leverage and risk-weighted assets and low Tier 1 capital have more exposure to risky sovereign debt, especially in

[30] See also Fact 1 in the next section for further details.

countries hit by the debt crisis.[31] It is important to note that these channels are not mutually exclusive. In fact, a weak form of moral suasion, whereby the government purposefully neglects to regulate against risky domestic sovereign bond purchases, is conducive to gambling. Uhlig (2014), Farhi and Tirole (2017) and Crosignani (2015) discuss the optimality of this from the domestic government's perspective.[32]

Finally, this chapter is related to two recent studies on central bank liquidity provision in the context of the European sovereign debt crisis. Drechsler et al. (2016) show that lender of last resort loans were mainly taken by under-capitalized banks and used for purchases of risky sovereign debt. Crosignani et al. (2016) show that the longer-term refinancing operations (LTRO) adopted by the European Central Bank (ECB) induced Portuguese banks to increase their holdings of risky domestic sovereign bonds. In the gambling equilibrium of this model, (unconditional) liquidity provision leads to a similar outcome.

Layout The remainder of this chapter is structured as follows: Section 2.2 presents the key stylized facts about the European sovereign debt crisis. Section 2.3 describes the model environment. Section 2.4 describes the propagation of sovereign risk shocks and examines the fit of the model to Portuguese data. Section 2.5 conducts policy analysis. Section 2.6 concludes.

2.2 Facts

In this section, I present four key stylized facts about the European sovereign debt crisis and the ensuing sovereign-bank nexus. I focus on five countries that were hit by the crisis, Greece, Ireland, Italy, Portugal and Spain (periphery), and contrast them with Germany (core), as a benchmark.

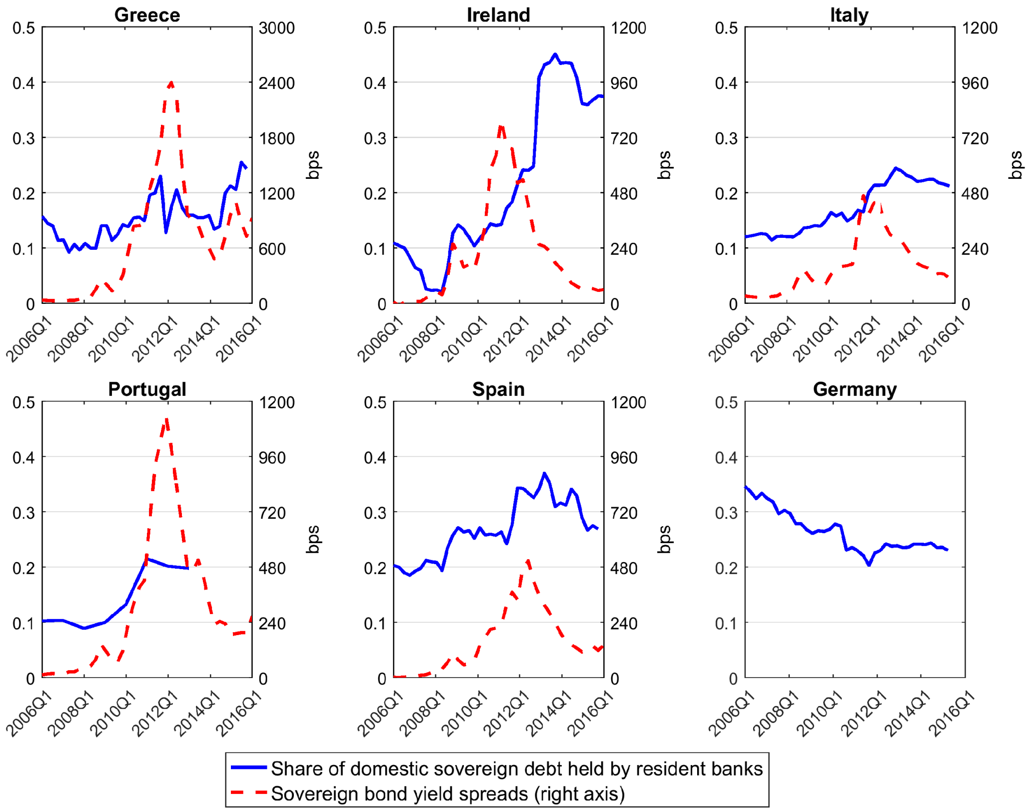
Fact 1. *In the periphery, the share of domestic sovereign debt held by the national banking system has sharply increased.*

Figure 2.1 shows that the yield spreads between sovereign bonds issued by the periphery countries and Germany (as a benchmark for safe assets) increase sharply after 2009 and peak in 2012. Thereafter, the spreads decrease but remain higher than their pre-crisis levels. Concurrently, there is an increase in the share of domestic government debt held by banks resident in these countries. In contrast, there is a decrease in the share of German sovereign debt held by German banks.

[31] See also Fact 2 in the next section.

[32] In the context of the Euro area, gambling is faciliated by the zero risk-weight attached to sovereign bonds issued by European Union member states in capital regulation (Bank for International Settlements, 2013).

Figure 2.1: **Sovereign bond holdings and yield spreads**



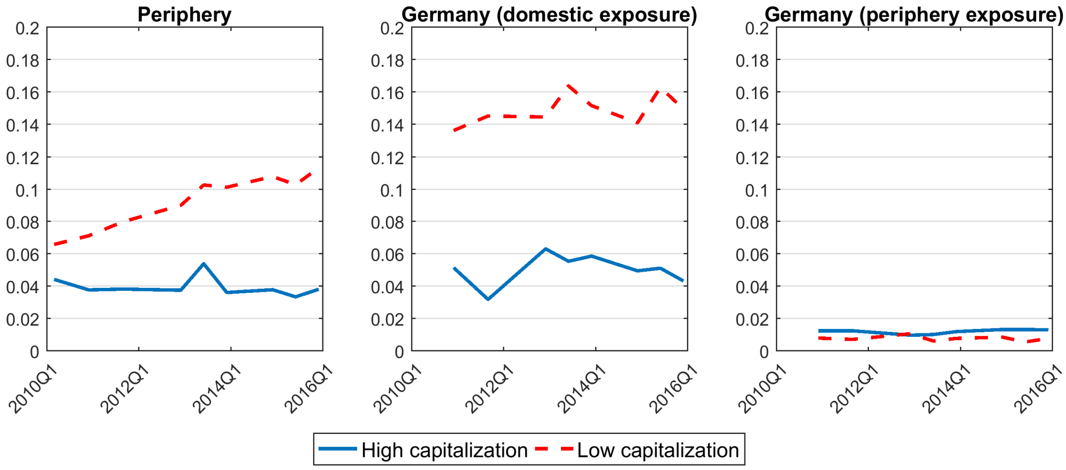
Note: Sovereign bond yields refer to bonds with 10 year maturity. Spreads are from German sovereign bond yields. Portuguese data on bond holdings is only available until 2012 and on an annual basis. All other data is quarterly. Source: OECD (MEI) and Merler and Pisani-Ferry (2012).

Fact 2. *Under-capitalized banks in the periphery have increased their exposure to domestic sovereign debt, while the exposures of well-capitalized banks in the periphery and German banks have remained nearly constant.*

The first panel of Figure 2.2 shows that the average domestic sovereign exposure of under-capitalized banks in the periphery has nearly doubled over 2010-2016, while that of capitalized banks remained near constant. This indicates a negative relationship between bank capitalization and the change in domestic sovereign debt exposures over the debt crisis.[33] The second panel shows that, in contrast to the periphery, domestic sovereign bond exposures of German banks with low and high capitalization do not follow a measurably different pattern over the crisis. This is also true for their exposure to bonds issued by peripheral countries as shown in the last panel. Thus, there is no apparent relationship between bank capitalization and changes in sovereign exposures for banks based in Germany.

[33] For an empirical analysis, see Acharya and Steffen (2015). They reach the same conclusion with a regression that controls for bank and country characteristics.

Figure 2.2: **Bank capitalization and sovereign exposures**



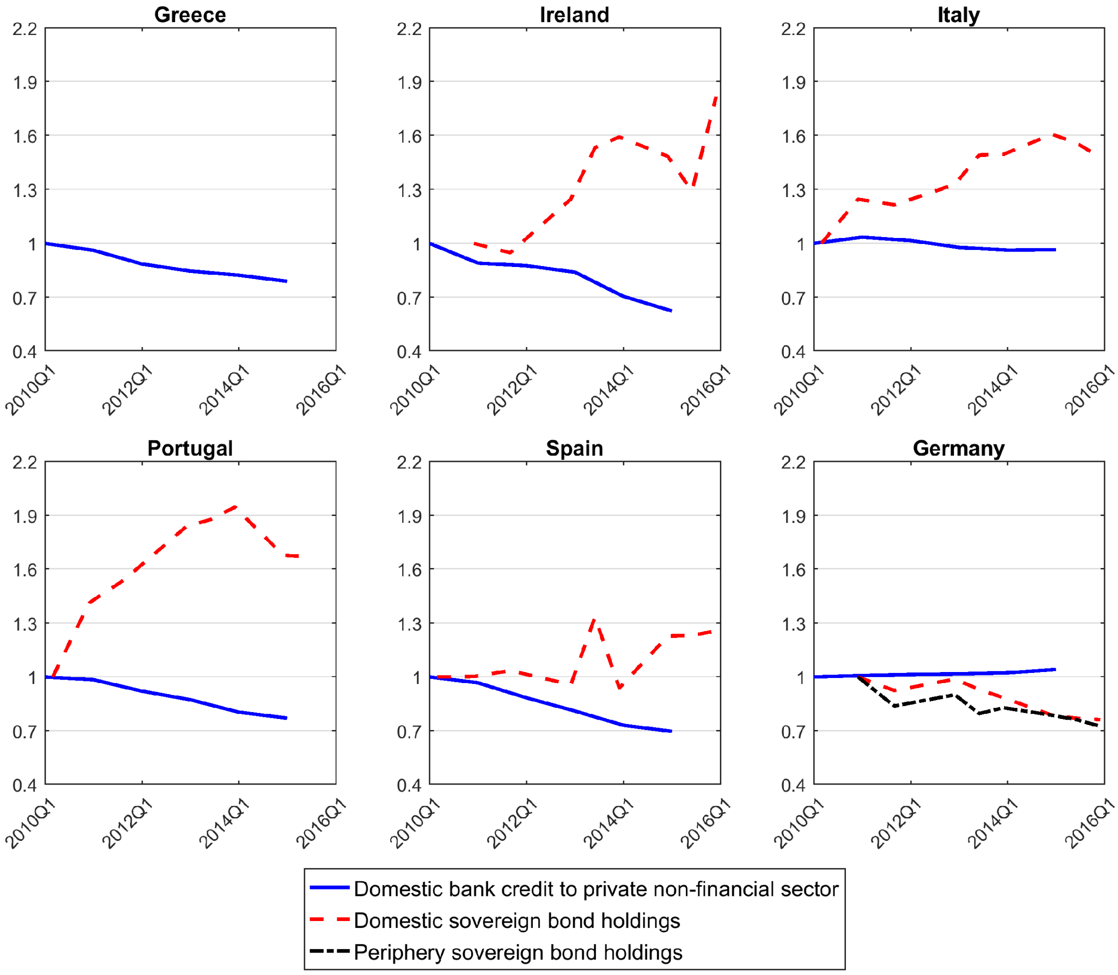
Note: Sovereign bond exposure refers to the share of sovereign bonds within total assets. No data is available for Greek banks. Low capitalization refers to banks with a Tier 1 Capital ratio below the first quartile in 2009. High capitalization refers to those above the third quartile. Source: Bloomberg and the European Banking Authority.

Together, these two findings lend support to the gambling hypothesis which suggests that under-capitalized banks based in default-risky countries have a specific incentive to purchase their own government's debt. This is due to the combination of limited liability with the anticipation of balance sheet costs independent to their sovereign bond holdings in the case of their own government's default. The latter aspect makes domestic sovereign bonds particularly suitable for risk shifting, since they yield a high return in the states of nature where banks have better solvency prospects.[34]

Contrast this with a mechanism that suggests the increase in domestic sovereign bond purchases is driven solely by limited liability. Under the regulatory framework present in the Euro area, sovereign bonds issued by all European Union member states carry zero risk-weight in capital regulation (Bank for International Settlements, 2013). Therefore, if limited liability was the sole driving factor, under-capitalized German banks would also have an incentive to purchase periphery sovereign debt. This would in turn lead to a negative relationship between bank capitalization and periphery exposure in Germany, which is not observed in Figure 2.2. In a similar vein, creditor discrimination effects where the expectation of selective default leads to the repatriation of risky sovereign debt, would lead to an increase in domestic sovereign exposure of periphery banks regardless of their capitalization. This is also not observed in

[34] In the case of sovereign default, gambling banks do not internalize the complete extent of the haircut on domestic sovereign bonds since they are protected by limited liability.

Figure 2.3: **Bank lending**



Note: Sovereign bond holdings are attained using data from EU-wide stress tests and transparency exercises. There is no data available for Greek banks. Domestic bank credit to private non-financial sector refers to financial resources provided to the private non-financial sector by domestic banks that establish a claim for repayment. Source: World Bank and the European Banking Authority.

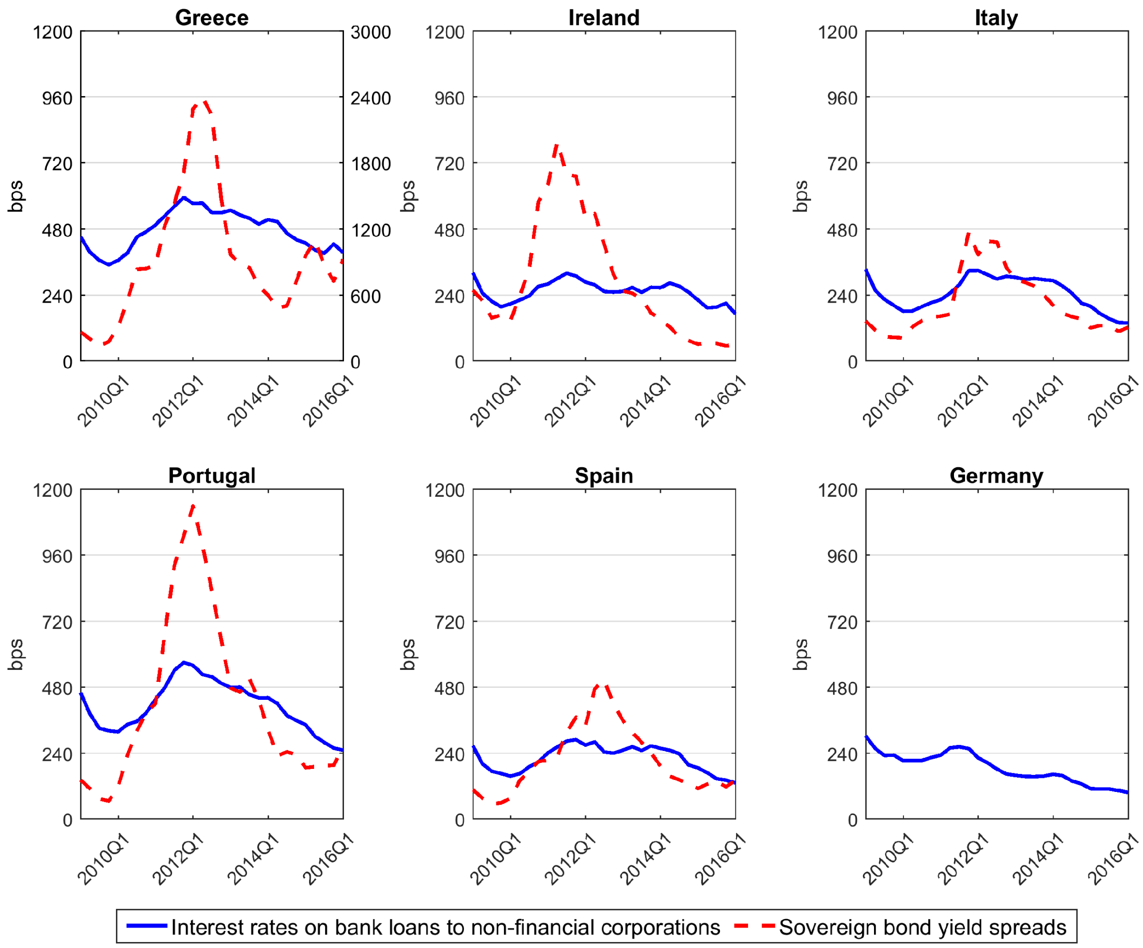
Figure 2.2.[35]

Fact 3. *In the periphery, banks reduced their lending to the private non-financial sector while increasing their domestic sovereign bond holdings. At the same time, there was a rise in private borrowing costs.*

Figure 2.3 shows that the volume of domestic sovereign bonds held by the national banking sector has increased by varying degrees in the periphery, ranging from about 30% in Spain to

[35]The patterns in Figure 2.2 are also compatible with the moral suasion hypothesis under the condition that risky governments can exert greater pressure on under-capitalized banks to purchase domestic sovereign debt. Note that the gambling and moral suasion hypotheses are not mutually exclusive. In fact, a "weak" form of moral suasion where the government neglects to regulate the domestic sovereign exposure of local banks is conducive to gambling. The optimality of this from the risky government's perspective is analysed by Crosignani (2015), Farhi and Tirole (2017) and Uhlig (2014).

Figure 2.4: **Loan interest rates**



Note: Loan interest rates refer to loans of all amounts by domestic banks to non-financial corporations (new business). Source: ECB.

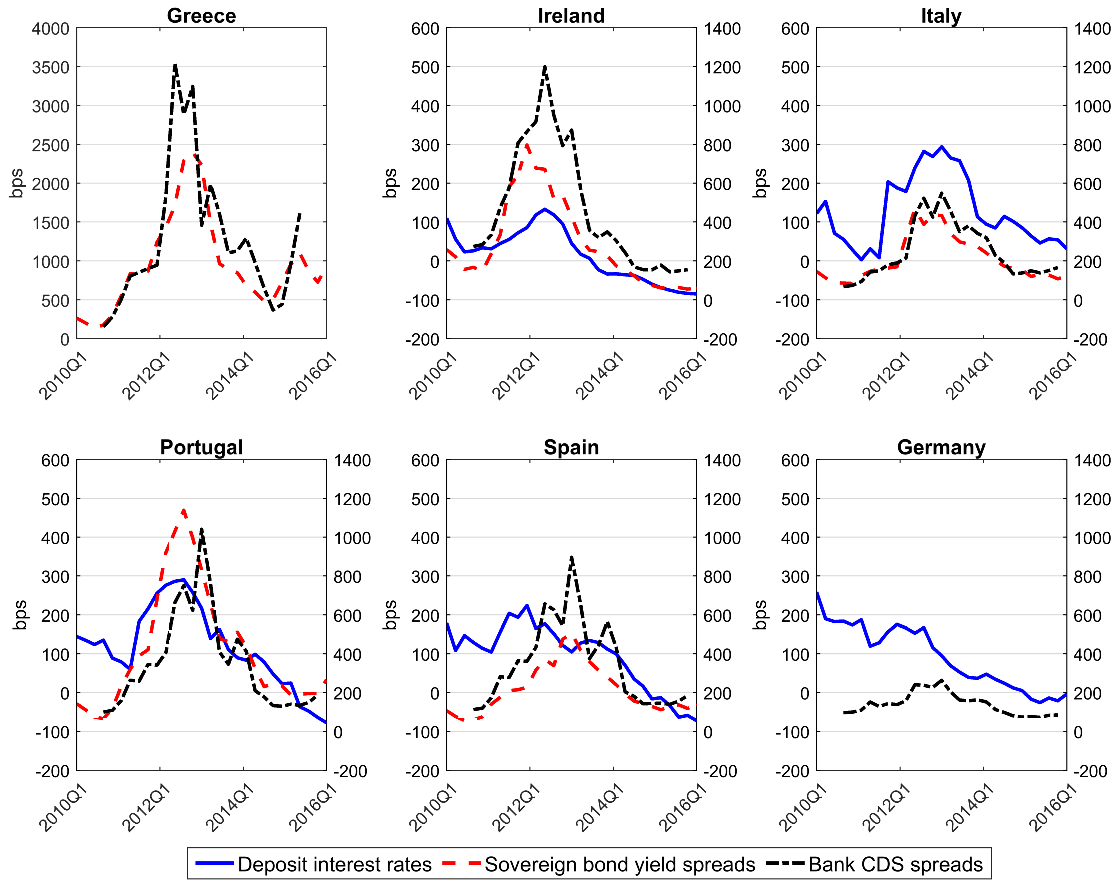
nearly double its initial amount in Ireland and Portugal. At the same time, credit to the private sector by domestic banks decreased by up to 30% in each periphery country except for Italy where it stagnated. Figure 2.4 shows that interest rates on loans to non-financial corporations also increased at the peak of the debt crisis in 2011-2012, especially in Portugal and Greece.

In Germany, on the other hand, banks reduced their holdings of both domestic and periphery sovereign bonds, and slightly increased their lending to the private sector. There was also a significant improvement in borrowing conditions faced by private non-financial corporations, with a decline of over 200 basis points in loan interest rates between 2010-2016.

A mechanism that can generate patterns similar to those present in Figures 2.3 and 2.4 is the crowding out of bank lending by domestic sovereign bond purchases.[36]

[36]For further empirical evidence on the effects of the sovereign debt crisis on credit to the private sector, see Acharya et al. (2014b), Becker and Ivashina (2014), De Marco (2017) and Popov and Van Horen (2015).

Figure 2.5: **Bank funding costs**



Note: The left axis represents deposit interest rates and the right axis represents bank CDS and sovereign bond yield spreads. Both axes are in basis points. Deposit interest rates refer to time deposits of all agreed maturities and amounts (new business). Bank CDS spreads refer to the implied CDS spread measure in Bloomberg. There is no available deposit interest rate data for Greece. Source: Bloomberg, ECB, OECD.

Fact 4. *There is substantial co-movement between sovereign bond yield spreads and bank funding costs in the periphery.*

Figure 2.5 plots bank credit default swap (CDS) spreads and deposit interest rates against sovereign bond yield spreads and Table 2.1 reports the corresponding correlation coefficients. The CDS spreads co-move significantly with sovereign spreads in the periphery, consistent with the notion of a sovereign-bank nexus where the solvency prospects of the government and the banking sector are intertwined.[37] To a lesser extent, deposit interest rates also move with yield spreads, especially during the peak of the crisis in 2011-2012. A potential explanation for this is that depositors expect a decline in the real value of their deposits in the event that the banking sector and government are both in default.

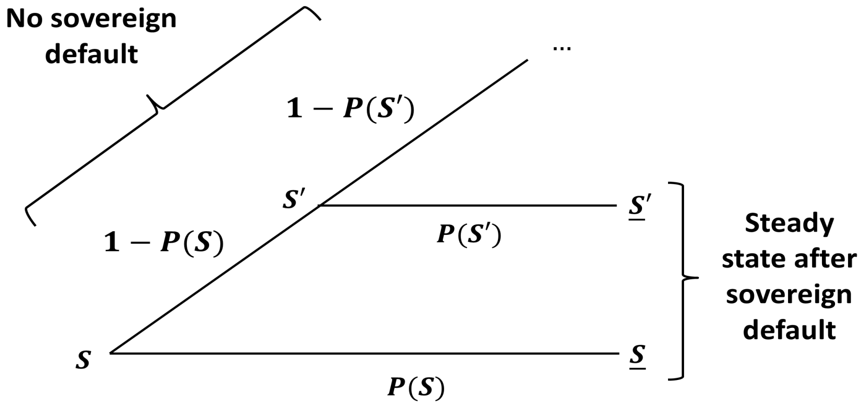
[37] Acharya et al. (2014a) show that changes in sovereign CDS explain changes in bank CDS even after controlling for aggregate and bank-level determinants of credit spreads.

h]

Table 2.1: **Correlation with sovereign bond yield spreads over 2010-2015**

	Greece	Ireland	Italy	Portugal	Spain
Bank CDS spreads	0.85	0.93	0.93	0.85	0.93
Deposit interest rates	–	0.84	0.84	0.74	0.37

Figure 2.6: **Recursive timeline**



In the next sections, I show that gambling on domestic sovereign debt can arise as an equilibrium outcome when banks are under-capitalized. In this gambling equilibrium, bank lending is crowded out by domestic sovereign bond purchases and bank funding costs co-move with domestic sovereign bond yields, consistent with the stylized facts described here.

2.3 Model environment

I consider a small open economy populated with three private agents: households, banks and firms, and a government issuing default-risky debt. In each period, banks collect deposits from households and use these funds, along with their own net worth, for domestic sovereign bond purchases and working capital lending to firms. Firms use working capital and labour rented from households to produce the consumption good.

Figure 2.6 shows the recursive timeline. The vector $\mathbf{S}$ collects the values of aggregate state variables (to be defined explicitly later on) in the current period and $\mathbf{S}'$ denotes the state vector for the next period. Sovereign default is incorporated into the model as an absorbing state. In each period, the government defaults with probability $P(\mathbf{S})$. Once the government defaults, there is no more sovereign default risk in future periods and the model economy moves to a steady state $\underline{S}$.

Sovereign default reduces the productivity of firms. This reflects the costs of domestic sovereign default on bank balance sheets, which hit them independently of their sovereign bond holdings.[38] As a result, banks receive a low return from their lending to firms as well as their domestic sovereign bond holdings.

When banks have sufficient funds to pay the promised return to their depositors, they remain solvent and accumulate a portion of their profits as net worth to use in the next period. Otherwise, they become insolvent under limited liability and a haircut proportionate to their funding shortfall is imposed on deposits.

The remainder of the section is organized as follows: First, I describe the process for sovereign risk, and the optimal strategies of firms, households and banks. Then I discuss the formulation of household sentiments, define the equilibrium concept and characterize the steady state after sovereign default. Finally, I provide a sketch of the algorithm used for the numerical solution.

2.3.1 Government

The government issues discount bonds B at a price $q^b(\mathbf{S})$. Sovereign bonds are internationally traded and their marginal buyers are deep pocketed foreign investors. As such, they are priced at their expected return

$$q^b(\mathbf{S}) = \left(1 - P(\mathbf{S}) + P(\mathbf{S})\,\theta^b\right) q^* \tag{2.1}$$

where $\theta^b \in (0,1)$ is their recovery rate and q^* is the price of an international safe asset d^* with perfectly elastic supply. In a monetary union, $1/q^*$ can be interpreted as the interest rate set by the common central bank.

The law of motion for government debt is given by the government's budget constraint

$$q^b(\mathbf{S})\,B' = B + G(\mathbf{S}) - T(\mathbf{S})$$

where $T(\mathbf{S})$ is lump-sum taxation on households and $G(\mathbf{S})$ is government spending. Since B has no effect on the non-government sector under this specification, the only restriction I place on the primary surplus $G(\mathbf{S}) - T(\mathbf{S})$ is that it follows a fiscal rule that precludes Ponzi games.

The sovereign default probability $P(\mathbf{S})$ is determined by a stochastic fiscal limit. Let $\Upsilon(\mathbf{S})$ denote the fiscal stress faced by the government. At the beginning of each period, an i.i.d. shock ε that follows a standard logistic distribution determines the government's resolve to avoid default. Sovereign default occurs when $\varepsilon \leq \Upsilon(\mathbf{S})$. The default probability is then given by the logistic function

$$P(\mathbf{S}) = \frac{\exp(\Upsilon(\mathbf{S}))}{1 + \exp(\Upsilon(\mathbf{S}))} \tag{2.2}$$

[38] For other studies which rely on output costs of default, see e.g. Cole and Kehoe (2000), Arellano (2008) and Aguiar et al. (2015).

The stock of government debt B, output Y and the sovereign bond yield $1/q^b(\mathbf{S})$ may easily be incorporated into the fiscal stress function $\Upsilon(\mathbf{S})$ as determinants of sovereign risk. For the dynamic solution, however, I adopt a simple specification $\Upsilon(\mathbf{S}) = \delta$ where δ follows the AR(1) process

$$\delta' = \delta_{ss} + \rho_\delta(\delta - \delta_{ss}) + \sigma_\delta\varepsilon'_\delta, \quad \varepsilon'_\delta \sim N(0,1) \tag{2.3}$$

and ε'_δ is a sovereign risk shock.

My reasons for adopting this specification are threefold. First, Borensztein and Panizza (2009) and Reinhart and Rogoff (2009) find that sovereign defaults are often accompanied with banking crises while Yeyati and Panizza (2011) attribute a large portion of the output costs of default to anticipation effects that precede the default event itself. Motivated by this empirical evidence, I focus on the financial interactions that take place under sovereign default risk and abstain from an explicit treatment of the processes that drive governments to default on their debt, which may include a range of economic and political factors.[39]

Second, by adopting this specification I abstain from the feedback loops between sovereign default risk and domestic fundamentals such as the stock of debt and sovereign bond yields. Although these feedback loops play a potentially important role in the transmission of sovereign risk, they have been studied extensively in recent literature (see e.g. Corsetti et al., 2013). Abstaining from them permits me to isolate the propagation channel of sovereign risk through bank-depositor interactions.

Third, recent empirical studies show that a substantial portion of the movements in sovereign risk premia during the recent sovereign debt crisis were unrelated to country fundamentals (see e.g. Bahaj, 2014; De Grauwe and Ji, 2012). In line with these findings, the sovereign risk shock (2.3) can be interpreted as reflecting non-fundamental factors such as contagion and self-fulfilling sentiments in sovereign bond markets.

2.3.2 Firms

Firms borrow working capital in advance of the sovereign default realization by issuing loans $L(\mathbf{S})$ to banks at a price $q^l(\mathbf{S})$. Working capital depreciates fully such that its evolution is given by

$$K(\mathbf{S}) = q^l(\mathbf{S})\,L(\mathbf{S}) \tag{2.4}$$

To produce the consumption good Y, firms combine working capital K and labour H hired from households with a standard Cobb-Douglas production technology. The representative

[39] See also Broner et al. (2014), Bocola (2016) and Brunnermeier et al. (2016) for other studies which analyse the financial effects of sovereign default without explicitly modelling the causes thereof.

firm's first order conditions are then given by

$$w(\mathbf{S}) = (1-\alpha) K(\mathbf{S})^{\alpha} \tag{2.5}$$

$$\underline{w}(\mathbf{S}) = (1-\alpha) \underline{A} K(\mathbf{S})^{\alpha} \tag{2.6}$$

$$q^{l}(\mathbf{S}) = \left(\frac{1}{\alpha A}\right)^{\frac{1}{\alpha}} L(\mathbf{S})^{\frac{1-\alpha}{\alpha}} \tag{2.7}$$

$$\theta^{l} = \frac{\underline{A}}{A} \tag{2.8}$$

where $\left(A, w(\mathbf{S}), \theta^{l}\right)$ are respectively productivity, wage, and the recovery rate of loans. An underbar denotes the state with sovereign default and the labour supply is perfectly inelastic and normalized to 1.

For future reference, note also that imperfectly competitive banks perceive (2.7) as

$$q^{l}(l, \mathbf{S}) = \left(\frac{1}{\alpha A}\right)^{\frac{1}{\alpha}} (l + (1-\phi) L(\mathbf{S}))^{\frac{1-\alpha}{\alpha}} \tag{2.9}$$

which internalizes the impact of their individual lending l on the price of loans, where ϕ is the market share of an individual bank such that $l = \phi L(\mathbf{S})$ in a symmetric equilibrium.

2.3.3 Households

Households have risk averse preferences with their flow utility $u(c)$ given by a standard CRRA specification. The representative household's problem can be written as

$$v^{h}(D, D^{*}; \mathbf{S}) = \max_{c, D', D^{*\prime}} \left\{ \begin{array}{c} u(c) + \beta(1 - P(\mathbf{S})) \mathbb{E}_{\mathbf{S}}\left[v^{h}(D', D^{*\prime}; \mathbf{S}')\right] \\ +\beta P(\mathbf{S}) \underline{v}^{h}(D', D^{*\prime}; \mathbf{S}') \end{array} \right\}$$

subject to

$$\begin{aligned} c + qD' + q^{*} D^{*\prime} &= D + D^{*} - T(\mathbf{S}) + w(\mathbf{S}) \\ \mathbf{S}' &= \mathbf{\Gamma}(\mathbf{S}) \end{aligned} \tag{2.10}$$

where q is the price of (domestic) bank deposits D', $D^{*\prime}$ represents safe assets, $\Gamma(.)$ is the law of motion for the aggregate state variables, and $\underline{v}^{h}(.)$ represents the household's continuation value under sovereign default. Lemma 2.1 provides an expression for $\underline{v}^{h}(.)$.

Lemma 2.1 *The continuation value for households in the steady state $\underline{S}$ is*

$$\underline{v}^h\left(D', D^{*\prime}; \mathbf{S}\right) = \frac{1}{1-\beta} u\left(\underline{c}\right) \ ,$$
$$\underline{c} = \left(1-q^*\right)\left(\theta D' + D^{*\prime} + \frac{1-\alpha}{\alpha}\frac{\underline{A}}{A} L\left(\mathbf{S}\right)\right) + q^* \underline{w} - \underline{T}$$

where $\underline{w}$ is given by

$$\underline{w} = \left(1-\alpha\right)\underline{AK^{\alpha}}$$

Proof. Provided in Appendix B2.1. ■

Observe that consumption $\underline{c}$ in the steady state is positively related to household wealth after sovereign default, which is increasing in the recovery rate θ of deposits. Using the above expressions, the first order conditions for risk-free assets D^* and domestic bank deposits D can be written as

$$\begin{aligned} q^* &= \beta \frac{\left(1-P\left(\mathbf{S}\right)\right)\mathbb{E}_{\mathbf{S}}\left[u_c\left(c'\right)\right] + P\left(\mathbf{S}\right) u_c\left(\underline{c}\right)}{u_c\left(c\right)} \\ q &= \beta \frac{\left(1-P\left(\mathbf{S}\right)\right)\mathbb{E}_{\mathbf{S}}\left[u_c\left(c'\right)\right] + P\left(\mathbf{S}\right) \theta u_c\left(\underline{c}\right)}{u_c\left(c\right)} \end{aligned}$$

where $u_c\left(.\right)$ is marginal utility.

The recovery rate anticipated by households depends on household expectations about the bank's domestic sovereign bond exposure $\tilde{\gamma}\left(n, \mathbf{S}\right)$ such that

$$\theta = \min\left\{1, \left(\tilde{\gamma}\left(n,\mathbf{S}\right)\frac{\theta^b}{q^b\left(\mathbf{S}\right)} + \left(1-\tilde{\gamma}\left(n,\mathbf{S}\right)\right)\frac{\theta^l}{q^l\left(\mathbf{S}\right)}\right)\left(\frac{n}{d'}+q\right)\right\}$$

where d' is deposits at bank level. The deposit demand schedule is attained by combining this expression with the household's first order conditions such that

$$q\left(d', n, \mathbf{S}\right) = \left\{ \begin{array}{ll} q^* & \text{for } d' \leq \bar{d}\left(n,\mathbf{S}\right) \\ q^* \dfrac{1-P(\mathbf{S}) + P(\mathbf{S})\frac{u_c(\underline{c})}{\mathbb{E}_{\mathbf{S}}\left[u_c(c')\right]}\left(\tilde{\gamma}(n,\mathbf{S})\frac{\theta^b}{q^b(\mathbf{S})} + (1-\tilde{\gamma}(n,\mathbf{S}))\frac{\theta^l}{q^l(\mathbf{S})}\right)\frac{n}{d'}}{1-q^* P(\mathbf{S})\frac{u_c(\underline{c})}{\mathbb{E}_{\mathbf{S}}\left[u_c(c')\right]}\left(\tilde{\gamma}(n,\mathbf{S})\frac{\theta^b}{q^b(\mathbf{S})} + (1-\tilde{\gamma}(n,\mathbf{S}))\frac{\theta^l}{q^l(\mathbf{S})}\right)} & \text{for } d' > \bar{d}\left(n,\mathbf{S}\right) \end{array} \right\}, \quad (2.11)$$

with the deposit threshold $\bar{d}\left(n,\mathbf{S}\right)$ defined as

$$\bar{d}\left(n,\mathbf{S}\right) = \frac{\left(\tilde{\gamma}\left(n,\mathbf{S}\right)\frac{\theta^b}{q^b(\mathbf{S})} + \left(1-\tilde{\gamma}\left(n,\mathbf{S}\right)\right)\frac{\theta^l}{q^l(\mathbf{S})}\right) n}{1-q^*\left(\tilde{\gamma}\left(n,\mathbf{S}\right)\frac{\theta^b}{q^b(\mathbf{S})} + \left(1-\tilde{\gamma}\left(n,\mathbf{S}\right)\right)\frac{\theta^l}{q^l(\mathbf{S})}\right)}$$

Finally, for future reference, I denote by $\mu_d(d', n, S)$ the mark-up banks enjoy in the deposit market

$$\begin{aligned}\mu_d(d', n, \mathbf{S}) &\equiv -\frac{\partial q(d', n, \mathbf{S})}{\partial d'}\frac{d'}{q(d', n, \mathbf{S})} \\ &= \left\{\begin{array}{ll} 0 & \text{for } d' \leq \bar{d}(n, \mathbf{S}) \\ \frac{P(\mathbf{S})\frac{u_c(\underline{c})}{\mathbb{E}_{\mathbf{S}}[u_c(c')]}\left(\tilde{\gamma}(n,\mathbf{S})\frac{\theta^b}{q^b(\mathbf{S})}+(1-\tilde{\gamma}(n,\mathbf{S}))\frac{\theta^l}{q^l(\mathbf{S})}\right)\frac{n}{d'}}{1-P(\mathbf{S})+P(\mathbf{S})\frac{u_c(\underline{c})}{\mathbb{E}_{\mathbf{S}}[u_c(c')]}\left(\tilde{\gamma}(n,\mathbf{S})\frac{\theta^b}{q^b(\mathbf{S})}+(1-\tilde{\gamma}(n,\mathbf{S}))\frac{\theta^l}{q^l(\mathbf{S})}\right)\frac{n}{d'}} & \text{for } d' > \bar{d}(n, \mathbf{S}) \end{array}\right\}\end{aligned}$$

which is increasing in the curvature of the deposit demand schedule, and hence the marginal utility wedge $\frac{u_c(\underline{c})}{\mathbb{E}_{\mathbf{S}}[u_c(c')]}$.

2.3.4 Banks

Each bank is managed by a unit continuum of risk-neutral bankers. From a representative bank's perspective, the timeline of events within a period is as follows. At the beginning of each period, the bank observes the realization of $\mathbf{S}$ and collects deposits d' from households at a price $q(d', n, \mathbf{S})$. It uses these deposits, along with its accumulated net worth n to purchase domestic sovereign bonds b and loans l from firms at prices $q^b(\mathbf{S})$ and $q^l(l, \mathbf{S})$, thereby selecting its sovereign exposure γ.[40]

Next, the bank learns whether the government is in default. The payoff from (b, l) and hence the bank's profits are contingent on the sovereign default realization

$$\pi = l + b - d' \tag{2.12}$$

$$\underline{\pi} = \max\left(\theta^l l + \theta^b b - d', 0\right) \tag{2.13}$$

such that the bank may be rendered insolvent by sovereign default. Bankers have limited liability, so when the bank becomes insolvent, all of its bankers exit the economy with zero payoff. When the bank is solvent, on the other hand, a randomly determined but constant portion $(1-\psi)$ of its bankers exit and consume their share of the profits.[41] The remaining

[40] As in Chapter 1, γ is related to (b, l) through the expressions $b = \gamma\left(\frac{n+q(d',n,\mathbf{S})d'}{q^b(\mathbf{S})}\right)$, $l = (1-\gamma)\left(\frac{n+q(d',n,\mathbf{S})d}{q^l(l,\mathbf{S})}\right)$

[41] The number of banks, and the bankers that manage them are constant over time. Insolvent banks are replaced with a new bank that has zero net worth. Bankers that exit from solvent banks are replaced with new bankers which do not contribute to net worth.

profits are accumulated as net worth in the next period, according to the law of motion

$$n' = \psi(\pi - \omega) \tag{2.14}$$
$$\underline{n}' = \psi(\underline{\pi} - \omega) \tag{2.15}$$

where ω represents overhead costs.[42]

Limited liability creates a discontinuity in the representative bank's policy function such that its decision problem can be written as a choice between two alternative strategies, a 'safe strategy' where the bank satisfies an occasionally binding solvency constraint

$$d' \leq \theta^l l + \theta^b b \tag{2.16}$$

and limited liability never binds, and a 'gambling strategy' which leaves the bank reliant on limited liability in the event of sovereign default. I denote these with the subscripts s and g.

The representative bank's problem can then be written as

$$\begin{aligned} v^b(n;\mathbf{S}) &= \max\left\{v^b_s(n;\mathbf{S}), v^b_g(n;\mathbf{S})\right\}, \\ v^b_s(n;\mathbf{S}) &= \max_{d',\gamma\in[0,1]}\left\{\begin{array}{c}(1-P(\mathbf{S}))\left((1-\psi)\pi + \psi\mathbb{E}_{\mathbf{S}}\left[v^b(n';\mathbf{S}')\right]\right) \\ +P(\mathbf{S})\left((1-\psi)\underline{\pi} + \psi\underline{v}^b(\underline{n}';\mathbf{S})\right)\end{array}\right\}, \\ v^b_g(n;\mathbf{S}) &= \max_{d',\gamma\in[0,1]}\left\{(1-P(\mathbf{S}))\left((1-\psi)\pi + \psi\mathbb{E}_{\mathbf{S}}\left[v^b(n';\mathbf{S}')\right]\right)\right\} \end{aligned} \tag{2.17}$$

subject to (2.12)-(2.15) and

$$\begin{aligned} q^b(\mathbf{S})\,b + q^l(l,\mathbf{S})\,l &= q(d',n,\mathbf{S})\,d' + n \\ \mathbf{S}' &= \mathbf{\Gamma}(\mathbf{S}) \end{aligned} \tag{2.18}$$

for both strategies, as well as the solvency constraint (2.16) for the safe strategy. $\mathbf{\Gamma}(\mathbf{S})$ is the law of motion for aggregate state variables, (2.18) represents the bank's budget constraint and $\underline{v}^b(.)$ is the bank's continuation value under sovereign default. Lemma 2.2 provides an expression for $\underline{v}^b(.)$.

Lemma 2.2 *The continuation value for solvent banks in the steady state $\underline{S}$ is*

$$\underline{v}^b(\underline{n}';S) = \underline{\pi} \tag{2.19}$$

[42] The consumption of portion $(1-\psi)$ of profits and overhead costs ω serve to prevent the accumulation of infinite net worth by banks in the steady state after sovereign default. The former aspect is standard in dynamic financial models while the latter is necessitated by the excess profits banks make due to imperfect competition. Overhead costs are waived when $\underline{\pi} < \omega$ so as to ensure that they never drive the bank into insolvency or affect the recovery rate θ on deposits.

Proof. Provided in Appendix B2.2. ■

The bank's first order conditions under the safe strategy are

$$\left(\theta^l l + \theta^b b - d'\right) \lambda\left(n, \mathbf{S}\right) = 0 \ , \ \lambda\left(n, \mathbf{S}\right) \geq 0 \ , \ d' \leq \theta^l l + \theta^b b \tag{2.20}$$

$$q^b\left(\mathbf{S}\right) \geq \frac{\left(1 - P\left(\mathbf{S}\right)\right)\left(1 - \psi + \psi \frac{\partial \mathbb{E}_{\mathbf{S}}\left[v^b\left(n', \mathbf{S}'\right)\right]}{\partial \pi}\right) + \left(P\left(\mathbf{S}\right) + \lambda\left(n, \mathbf{S}\right)\right)\theta^b}{1 + \lambda\left(n, \mathbf{S}\right)} \left(1 - \mu_d\left(d', n, \mathbf{S}\right)\right) q\left(d', n, \mathbf{S}\right) \tag{2.21}$$

$$\frac{q^l\left(l, \mathbf{S}\right)}{1 - \mu_l} = \frac{\left(1 - P\left(\mathbf{S}\right)\right)\left(1 - \psi + \psi \frac{\partial \mathbb{E}_{\mathbf{S}}\left[v^b\left(n', \mathbf{S}'\right)\right]}{\partial \pi}\right) + \left(P\left(\mathbf{S}\right) + \lambda\left(n, \mathbf{S}\right)\right)\theta^l}{1 + \lambda\left(n, \mathbf{S}\right)} \left(1 - \mu_d\left(d', n, \mathbf{S}\right)\right) q\left(d', n, \mathbf{S}\right) \tag{2.22}$$

where $\left(\mu_l, \mu_d\left(d', n, \mathbf{S}\right)\right)$ are the mark-ups in the loan and deposit markets and $\lambda\left(n, \mathbf{S}\right)$ is the Lagrange multiplier associated with the solvency constraint. The interpretation of these conditions is similar to their counterparts (1.24)-(1.26) in Chapter 1. The two sets of FOCs differ only due to the term $\frac{\partial \mathbb{E}_{\mathbf{S}}\left[v^b\left(n', \mathbf{S}'\right)\right]}{\partial \pi}$ which is the expected value of a marginal increase in profits in the state without sovereign default. In a two-period setting, this term is fixed at unity by the bank's risk neutrality. In a dynamic environment, on the other hand, it depends on the marginal value of net worth in future state realizations $\mathbf{S}'$ via (2.14). Proposition 2.1 shows that the FOCs in the first chapter constitute a special case of the dynamic FOCs.

Proposition 2.1 *Let $\mathfrak{g}$ be the subset of state realizations where the bank follows a gambling strategy. If for all possible future aggregate state realizations $\mathbf{S}'$, either $\left(n'; \mathbf{S}'\right) \in \mathfrak{g}$ or $\left(n'; \mathbf{S}'\right) \notin \mathfrak{g}$ and $\lambda\left(n', \mathbf{S}'\right) = 0$, $q\left(d', n', \mathbf{S}'\right) = q^*$, then*

$$\frac{\partial \mathbb{E}_{\mathbf{S}}\left[v^b\left(n', \mathbf{S}'\right)\right]}{\partial \pi} = 1$$

Otherwise

$$\frac{\partial \mathbb{E}_{\mathbf{S}}\left[v^b\left(n', \mathbf{S}'\right)\right]}{\partial \pi} > 1$$

Proof. Provided in Appendix B2.3. ■

The proposition states that the bank attaches a higher value to future net worth if there is a positive probability of visiting a future state realization where it follows a safe strategy with a binding solvency constraint and/or its deposits are perceived to be risky. This increase in the value attached to π relative to $\underline{\pi}$ increases the risk-taking incentives of the bank, leading to a rise in $\left(b, d'\right)$ under the safe strategy when the solvency constraint is slack, as well as stronger incentives to adopt the gambling strategy.

In contrast, the FOCs under the gambling strategy are identical to their counterparts in Section 1.2.6.

$$q^{b}(\mathbf{S}) = (1-\mu_{d}(d',n,\mathbf{S}))\,q(d',n,\mathbf{S}) \tag{2.23}$$

$$q^{l}(l,\mathbf{S}) = (1-\mu_{l})\,q^{b}(\mathbf{S}) \tag{2.24}$$

This is due to the bank's reliance on limited liability under sovereign default. Because of this, the bank only internalizes its profits π in the absence of sovereign default. Since the relative valuation of $(\pi, \underline{\pi})$ does not matter, the term $\frac{\partial \mathbb{E}_{\mathbf{S}}\left[v^{b}(n',\mathbf{S}')\right]}{\partial \pi}$ drops out of the gambling FOCs. In other words, when a bank follows the gambling strategy, its optimal set of actions are those that maximize π regardless of its time horizon.

2.3.5 Sentiments and sunspots

In this section, I describe how households formulate their expectations $\tilde{\gamma}(n,\mathbf{S})$ about a bank's domestic sovereign bond exposure. Conditional on $(n,\mathbf{S})$, the bank's first order conditions (2.20)-(2.24) provide a unique mapping between the strategy followed by a bank and its sovereign exposure.

Using (2.17), the optimality condition for the bank to adopt a gambling strategy can be written as

$$v_{g}^{b}(n;\mathbf{S}) \geq v_{s}^{b}(n;\mathbf{S}) \tag{2.25}$$

When this condition is satisfied, the bank's optimal exposure γ_g is given by (2.23), (2.24). Otherwise, the bank adopts a safe strategy and its exposure γ_s is pinned down by (2.20)-(2.22). Sentiments may become self-fulfilling due to the dependence of both sides of the inequality in (2.25) on $\tilde{\gamma}(n,\mathbf{S})$.

The state space for $(n,\mathbf{S})$ can be segmented into three non-intersecting subsets according to the interaction between (2.25) and $\tilde{\gamma}(n,\mathbf{S})$. Let $\mathcal{G}$ denote a subset where (2.25) is satisfied for $\tilde{\gamma}(n,\mathbf{S}) = \left\{\gamma_g, \gamma_s\right\}$, $\mathcal{S}$ denote a second subset where (2.25) is violated for $\tilde{\gamma}(n,\mathbf{S}) = \left\{\gamma_g, \gamma_s\right\}$ and $\mathcal{M}$ denote a third subset where (2.25) is satisfied for $\tilde{\gamma}(n,\mathbf{S}) = \gamma_g$ and violated for $\tilde{\gamma}(n,\mathbf{S}) = \gamma_s$. In the first two subsets $\{\mathcal{G},\mathcal{S}\}$, γ is uniquely determined regardless of $\tilde{\gamma}(n,\mathbf{S})$ while household sentiments become self-fulfilling when $(n,\mathbf{S}) \in \mathcal{M}$.

I resolve the multiplicity in $\mathcal{M}$ with the use of sunspots. Specifically, let ζ be a random variable drawn from a uniform distribution on the unit interval at the beginning of each period and $\bar{\zeta} \in [0,1]$ a constant threshold. When $\zeta > \bar{\zeta}$ household expectations coordinate on $\tilde{\gamma}(n,\mathbf{S}) = \gamma_s$ consistent with the safe strategy. I refer to this as 'good sentiments'. When $\zeta \leq \bar{\zeta}$, on the other hand, expectations coordinate on $\tilde{\gamma}(n,\mathbf{S}) = \gamma_g$ in line with the gambling strategy and there are 'bad sentiments'. To provide a formal definition for $\tilde{\gamma}(n,\mathbf{S})$, $\mathcal{M}$ is further segmented into two subsets $\mathcal{M}^+$ and $\mathcal{M}^-$ which respectively denote good and bad sentiments such that

$$\tilde{\gamma}(n, \mathbf{S}) = \begin{cases} \gamma_g \text{ if } (n, \mathbf{S}) \in \{\mathcal{G}, \mathcal{M}^-\} \\ \gamma_s \text{ if } (n, \mathbf{S}) \in \{\mathcal{S}, \mathcal{M}^+\} \end{cases}$$

Since ζ is uniformly distributed on a unit interval, the probability of good and bad sentiments in the next period are simply given by $\left(1-\bar{\zeta}\right)$ and $\bar{\zeta}$ respectively. Note that it is straightforward to introduce a more sophisticated specification for sunspots by replacing $\bar{\zeta}$ with an AR(1) shock process or a function of fundamentals such as the recovery rate θ of domestic deposits or government debt B. I opt for this simple specification as it permits me to isolate the role of sovereign risk and other relevant fundamentals in making household sentiments self-fulfilling in the first place. The subset $\mathcal{M}$ which provides a mapping of states with multiplicity is endogenously determined by the optimal strategies of households and banks, which in turn depend on these fundamentals.[43]

2.3.6 Steady state after sovereign default

When the government defaults, sovereign bond holders receive a recovery rate $\theta^b < 1$. Productivity also declines to $\underline{A} < A$ which leads to a reduction in wages and a partial payment from loans. If the banks followed a gambling strategy before sovereign default, they become insolvent such that households receive a recovery rate θ from their deposits and the banking sector is replaced with a new set of banks with zero net worth. Otherwise, deposits are repaid fully and bank net worth is determined by (2.15).

In the following period, the economy immediately moves to a steady state $\underline{S}$ where productivity recovers back to A and there is no further sovereign default risk.[44] In the absence of bank insolvency risk, domestic deposits become perfectly substitutable with risk-free assets

[43] Global games constitutes an alternative approach to sunspots in resolving multiple equilibria that creates an endogenous relationship between economic fundamentals and equilibrium selection. This approach, however, is not implementable in the context of the multiplicity considered in this chapter since the strategic complementarity is between banks and households, and takes place through a market mechanism that is capable of aggregating diverse beliefs. To see this, consider the introduction of a private signal to households about $\tilde{\gamma}(n, \mathbf{S})$. Provided households are not extremely risk averse, the solvency calculus of a household is not affected by the signal received by other households. Banks then find it optimal to borrow solely from the household with the lowest $\tilde{\gamma}(n, \mathbf{S})$ signal, which determines the price $q(d', n, \mathbf{S})$ in deposit markets. The model collapses to a sunspot solution where the lowest $\tilde{\gamma}(n, \mathbf{S})$ signal becomes the de facto sunspot.

[44] The immediate recovery in productivity only serves to simplify the exposition. This can be replaced with any continuation path for productivity as long as there is perfect foresight about it.

such that $\underline{q} = q^*$.[45] The steady state price and quantity of loans is then given by

$$\underline{q}^k = (1 - \mu_l)\, q^* \tag{2.26}$$
$$\underline{L} = (\alpha A)^{\frac{1}{1-\alpha}} \left(\underline{q}^k\right)^{\frac{\alpha}{1-\alpha}} \tag{2.27}$$

The following parameterization for (ψ, ω, q^*) is necessary to ensure this

$$\psi = q^* = \beta$$
$$\omega = \phi \mu_l \underline{L}$$

The parameterization for (ψ, ω) ensures that bank net worth remains constant while equating the risk-free asset price to the household discount factor drives households to completely smooth their consumption after sovereign default.[46]

2.3.7 Equilibrium

Let $\mathbf{S} = [N, \delta, \zeta, \varkappa]$ be the aggregate state sector, where $N = n/\phi$ is aggregate bank net worth in equilibrium and $\varkappa \equiv D + D^* + w(\mathbf{S}) - T(\mathbf{S})$ is disposable household wealth. A recursive rational expectations equilibrium is given by value functions for households and banks $\{v^h, v^b\}$ and policy functions for households $\{c, D', D^{*\prime}\}$ and for banks $\{\gamma, d'\}$ such that, given prices $\{w, \underline{w}, q^*\}$ and price schedules $\{q^l, q\}$: **(i)** households' and banks' value and policy functions solve their optimization problems; **(ii)** the market for domestic deposits clears, $D' = d'/\phi$ **(iii)** the market for loans clears $L(\mathbf{S}) = l/\phi$; **(iv)** the government budget constraint is satisfied; **(v)** $\mathbf{\Gamma}(.)$ and $\{\mathcal{G}, \mathcal{M}, \mathcal{S}\}$ are consistent with agents' optimal strategies.[47]

2.3.8 Numerical solution

The solution for the recursive equilibrium is attained using global numerical methods. In this section, I sketch the main steps in the algorithm and relegate the remaining details to Appendix B4.

Note that the decentralized, imperfectly competitive nature of banks requires the inclusion of individual bank net worth n along with $\mathbf{S}$ as a state variable. Specifically, although banks are

[45] There is no need take a stance on when and whether the government returns to sovereign bond markets as long as there is no further default risk. If the government is able to issue bonds, they are priced at $\underline{q}^b = q^*$ and banks are indifferent to holding them.

[46] Solving the household's problem when q^* differs from the discount factor β is trivial but leads to a balanced growth path for consumption rather than a steady state value. I abstain from this since it leads to additional complication without yielding any insights of interest.

[47] In the small open economy setting, the markets for goods and sovereign bonds are cleared through trade with foreign agents. Therefore, there is no need to explicitly include the clearing conditions for these markets in the equilibrium definition.

symmetric with net worth $n = \phi N$ on the equilibrium path, determining their optimal strategy as per Section 2.3.4 requires considering off-equilibrium strategies (deviations) which lead to a different path of n for the specific bank than the remainder of the banking sector. The bank's value function $v^b(n; \mathbf{S})$ and the equilibrium regions $\{\mathcal{G}, \mathcal{M}, \mathcal{S}\}$ are thus defined over $(n, \mathbf{S})$.

Let $X(\mathbf{S}) = \{\gamma, d', c, D', D^{*\prime}\}$ collect the policy functions of banks and households in the symmetric equilibrium with $n = \phi N$, and $\mathcal{E} = \{\mathcal{G}, \mathcal{M}, \mathcal{S}\}$ denote the equilibrium regions. The solution algorithm can then be sketched as follows

1. Begin with a set of guesses for $\{\mathcal{E}, \Gamma(\mathbf{S}), X(\mathbf{S})\}$.
2. Formulate future expectations according to $\{\mathcal{E}, \Gamma(\mathbf{S}), X(\mathbf{S})\}$. Then, use the deposit demand schedule in Section 2.3.3, first order conditions in Section 2.3.4, and the market clearing conditions in Section 2.3.7 to update $\{\Gamma(\mathbf{S}), X(\mathbf{S})\}$. Iterate until the solution for $\{\Gamma(\mathbf{S}), X(\mathbf{S})\}$ converges.
3. Guess the bank's value function $v^b(n; \mathbf{S})$.
4. Use the first order conditions in Sections (2.3.4) and (2.25) with expectations formulated according to $\{\mathcal{E}, \Gamma(\mathbf{S}), X(\mathbf{S})\}$ to update $v^b(n; \mathbf{S})$. Iterate until the solution for $v^b(n; \mathbf{S})$ converges.
5. Update $\mathcal{E}$ according to the solution to step 4. Repeat from step 2 until convergence.

I follow three distinct approaches to alleviate the curse of dimensionality that arises from solving the model globally. First, I use a piecewise cubic Hermite spline to interpolate $\{\Gamma(\mathbf{S}), X(\mathbf{S}), v^b(n; \mathbf{S})\}$ between the pre-defined grid points. Second, I abstain from the household's wealth accumulation process by letting lump-sum taxes $T(\mathbf{S})$ adjust to ensure that

$$\varkappa \equiv D + D^* + w(\mathbf{S}) - T(\mathbf{S}) = \bar{E}$$

as long as the government remains solvent, where $\bar{E}$ is a fixed wealth parameter. This does not affect households' incentives to save since they take $T(\mathbf{S})$ as given, but eliminates $\varkappa$ from the state vector, reducing the number of state variables to 4.

Third, I take advantage of a series of characteristics of the bank's first order conditions to reduce the computational burden in steps 2 and 4 significantly. Specifically, the FOCs (2.20) and (2.22) indicate that the optimal choices $\{\gamma_s, d'_s\}$ under a safe strategy are (i) independent of $\left\{\Gamma(\mathbf{S}), X(\mathbf{S}), v^b(n, \mathbf{S})\right\}$ when $\tilde{\gamma}(n, \mathbf{S}) = \gamma_s$ (ii) independent of $\left\{\Gamma(\mathbf{S}), v^b(n, \mathbf{S})\right\}$ when $\lambda(n, \mathbf{S}) > 0$. Similarly, the FOCs (2.23) and (2.24) indicate that the optimal choices $\left\{\gamma_g, d'_g\right\}$ under a gambling strategy are independent of $\left\{\Gamma(\mathbf{S}), v^b(n, \mathbf{S})\right\}$. The relevant proofs are provided in Appendix B4.

2.4 Numerical results

This section provides numerical results from the dynamic model under a calibration that targets Portugal. It proceeds in four steps. Section 2.4.1 describes the calibration. Section 2.4.2 discusses the relationship between sovereign risk and the equilibrium regions. Section 2.4.3 demonstrates the propagation of sovereign risk shocks with the use of impulse response functions to a sovereign risk shock. Finally, Section 2.4.4 brings the model to data by comparing its fit to the Portuguese economy over 2010-2016.

2.4.1 Calibration

The calibration targets Portugal over the crisis period of 2010-2016 with each period representing a quarter. Table 2.2 reports the calibrated parameters.

The recovery rate of sovereign bonds is set to $\theta^b = 0.6$ according to Cruces and Trebesch (2013). The calibration for the fiscal stress parameters $(\delta_{ss}, \rho_\delta, \sigma^2_\delta)$ matches $q^b(\mathbf{S})/q^*$ to the yield spread between Portuguese and German bonds (which act as a benchmark for the safe rate).[48] Specifically, I use (2.1) and (2.2) to back out a time series of fiscal stress realizations $\hat{\delta}_t$ from the spread data under the calibrated recovery rate. The calibration for $(\delta_{ss}, \rho_\delta, \sigma^2_\delta)$ is then attained by fitting the AR(1) process given by (2.3) to $\hat{\delta}_t$.[49]

In the household sector, the discount factor is calibrated to $\beta = 0.99^{1/4}$ and the wealth parameter targets data on household net worth from OECD. The calibration for the coefficient of risk aversion $\sigma = 3$ lies within the range given by recent empirical estimates (Thimme, 2016).

Regarding firms, I set the output elasticity of capital to the standard Cobb-Douglas value of $\alpha = 1/3$. In the absence of sovereign default, productivity is normalized to $A = 1$ such that $\underline{A}$ is equivalent to the recovery rate of loans θ^l. The calibration for $\underline{A}$ targets the recovery rate since sovereign default propagates through balance sheet costs to banks rather than the direct effects of productivity decline. Accordingly, I calibrate $\theta^l = 0.90$ in line with recent estimates on the effects of sovereign default on firm profitability (Hébert and Schreger, 2016).[50]

The bank market share parameter ϕ is calibrated to match the mark-up μ_l in the loans market to the average interest margin on domestic bank lending to non-financial corporations

[48] I use bonds with a remaining maturity of 3 months due to the quarterly calibration of the model. While the standard benchmark for measuring sovereign default risk is the yield/CDS spreads on 10 year bonds, it is not possible to extract quarter-on-quarter default probabilities from these measures without imposing additional restrictions on the yield curve.

[49] See Appendix B3 for further details.

[50] This implies a relatively high output cost of default compared to the previous literature. It is worth noting, however, that the calibration for θ^l can be reconciled with lower output costs with the introduction of bankruptcy costs or real frictions that limit the ability of firms to decrease salary costs following sovereign default. Note also that, under the baseline calibration, the parameter restrictions in (1.15) are satisfied for a wide range of recovery rates $\theta^l \in [0.59, 0.99]$. The qualitative results presented in Chapter 1, including the non-emptiness of the multiple equilibria region, remain valid at all points within this range.

Table 2.2: **Calibration**

Parameter	Value	Description	Source
θ^b	0.60	Sov. bond recovery rate	Cruces and Trebesch (2013)
δ_{ss}	-5.14	Fiscal stress (mean)	Bloomberg
ρ_δ	0.74	Fiscal stress (persistence)	Bloomberg
σ_δ^2	0.93	Fiscal stress (variance)	Bloomberg
β	$0.99^{1/4}$	Discount factor	-
$\bar{E}$	0.07E-9	Household wealth	OECD
σ	3.00	Coefficient of risk aversion	Thimme (2016)
α	0.33	Cobb-Douglas parameter	-
A	1.00	Productivity (no sov. default)	-
$\underline{A}$	0.90	Productivity (sov. default)	Hébert and Schreger (2016)
ϕ	0.005	Bank market share	ECB Statistical Data Warehouse
$\bar{\zeta}$	0.50	Probability of bad sentiments	-

during the pre-crisis period of 2003-2007.[51] Finally, I calibrate the sunspot threshold to $\bar{\zeta} = 0.5$ such that good and bad sentiments are equally likely.

2.4.2 Sovereign risk and equilibrium regions

I begin analysing the numerical results by examining the implications of sovereign risk for the equilibrium regions. Figure 2.7 provides a mapping of the prevalent equilibrium type across a range of sovereign default probabilities $P(\mathbf{S})$ and aggregate bank net worth N. As with the two period model in Section 1.2, the three equilibrium regions are ordered monotonically across net worth: First, the gambling equilibrium is unique when net worth falls short of a boundary $\underline{N}(\mathbf{S})$. Second, there is an intermediate multiplicity region $\underline{N}(\mathbf{S}) \leq N \leq \bar{N}(\mathbf{S})$. Finally, the safe equilibrium is unique when net worth exceeds $\overline{N}(\mathbf{S})$.

These boundaries are contingent on sovereign default risk. When sovereign bonds are completely safe, only a safe equilibrium is possible.[52] The emergence of sovereign risk, however,

[51] The relationship between the mark-up and the steady state price of loans is given by (2.26). I match this with pre-crisis interest rates in order to isolate the excess return due to market power.

[52] This stems from the lack of other types of aggregate risk within the model environment. It can, however, be interpreted as the reduced form outcome of a richer environment with capital regulation based on risk-weighted assets. In this environment, capital requirements faced by a bank depend on the risk-weight attached to its portfolio. For assets with non-sovereign risk, positive risk weights align the bank's incentives towards following a safe strategy. If sovereign bonds have zero risk-weight, Sovereign bonds, on the other hand, have a zero risk-weight, then gambling is only possible in the presence of sovereign default risk. The preferential treatment for sovereign bonds described here approximately reflects the regulatory framework in the Euro area (Bank for International Settlements, 2013).

creates a large region with a unique gambling equilibrium. Further increases in sovereign risk have a non-linear effect on banks' incentives to gamble. As sovereign risk rises, $\overline{N}(\mathbf{S})$ first increases, and then decreases while $\underline{N}(\mathbf{S})$ decreases throughout, leading to a widening of the multiplicity region.

To understand these findings, consider the implications of sovereign risk for bank payoffs under each strategy. When a bank follows the gambling strategy, a rise in sovereign risk has two opposing effects on its profits. First, it increases sovereign bond yields which raises profits from gambling. Second, it leads to a rise in bank funding costs which reduces profits. At low levels of sovereign risk, the former effect dominates such that a rise in $P(\mathbf{S})$ strengthens incentives to gamble. As bank funding costs are determined by risk averse households, however, the latter effect becomes stronger as sovereign risk increases. $\overline{N}(\mathbf{S})$ peaks at the point where the latter effect becomes dominant and the value associated with adopting the gambling strategy is negatively related to $P(\mathbf{S})$ beyond this point.

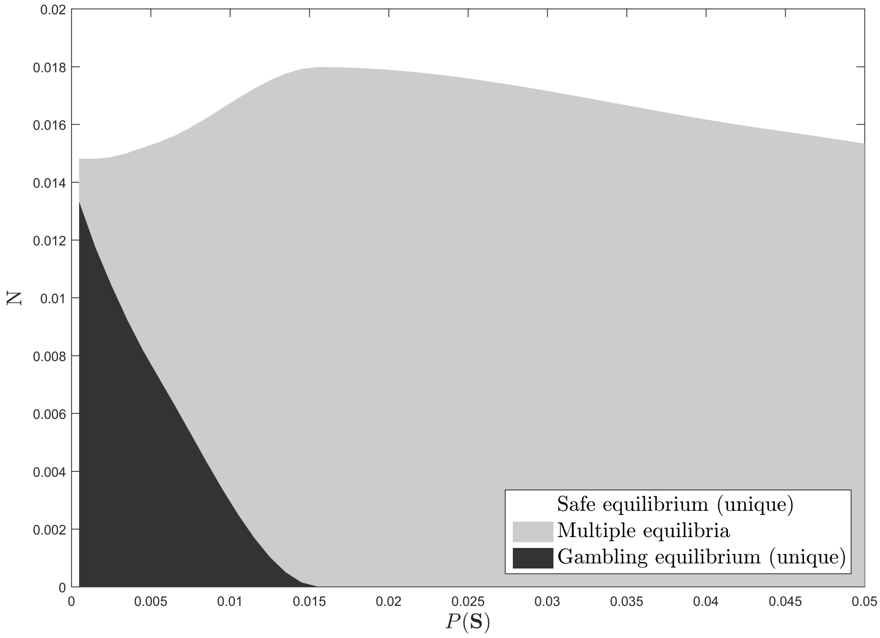
The impact of sovereign risk on the safe strategy payoff is contingent on household sentiments. Recall that the bank's solvency constraint coincides with its deposit threshold under good sentiments. This ensures that the bank borrows at the risk-free rate regardless of the sovereign default probability. As a result, the safe strategy payoff is largely independent of $P(\mathbf{S})$ when there are good sentiments.[53] Under bad sentiments, the deposit threshold becomes tighter than the solvency constraint due to the expectation of a high sovereign exposure. Despite following a safe strategy, banks optimally breach the deposit threshold such that households anticipate their insolvency under sovereign default. This leads to a positive relationship between bank funding costs and $P(\mathbf{S})$. The safe strategy payoff thus decreases in sovereign risk under bad sentiments.

This mechanism explains the widening of the multiplicity region as sovereign risk increases. A rise in $P(\mathbf{S})$ leads to a greater reduction in incentives to follow a safe strategy under bad sentiments than it does under good sentiments. This leads to the expansion of the region of net worth where both types of sentiments are self-fulfilling.

The pattern followed by the boundaries $\left(\overline{N}(\mathbf{S}), \underline{N}(\mathbf{S})\right)$ can also be explained by comparing the safe and gambling strategy payoffs. $\underline{N}(\mathbf{S})$ traces the levels of net worth where banks are indifferent between the two strategies under bad sentiments. Since the payoff from gambling first increases then falls in $P(\mathbf{S})$, while that of the safe strategy falls monotonically, $\underline{N}(\mathbf{S})$ declines sharply as sovereign risk increases. In contrast, $\overline{N}(\mathbf{S})$ traces the points of indifference under good sentiments, where the safe strategy payoff is independent of $P(\mathbf{S})$. Therefore, it has the same non-monotonic shape as the gambling payoff.

[53] To be precise, the payoff is independent of $P(S)$ when the solvency constraint is binding, which is the case at the boundary of net worth $\overline{N}(\mathbf{S})$. When the solvency constraint is slack, the expected payoff falls slightly as $P(\mathbf{S})$ increases due to a decline in bank lending.

Figure 2.7: **Equilibrium Mapping**



2.4.3 Propagation of sovereign risk shocks

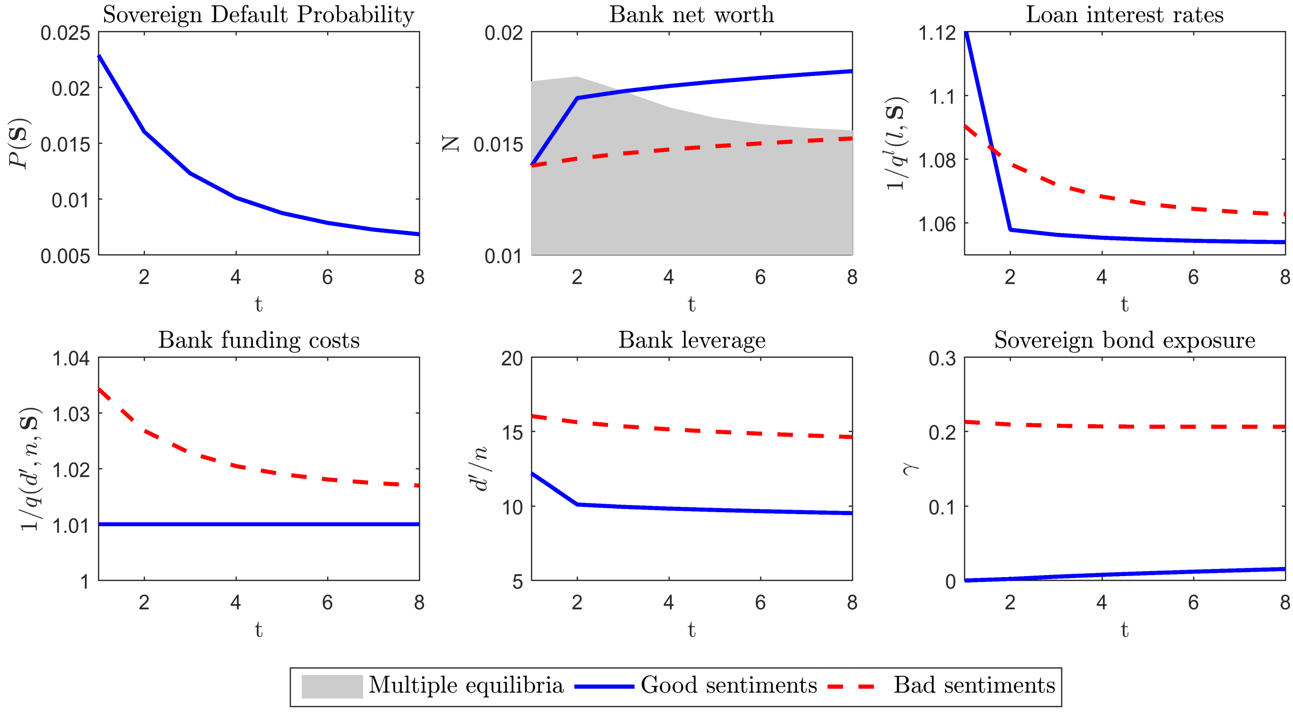
The next step is to evaluate the propagation of sovereign risk shocks. Figure 2.8 plots the response of key variables to an increase in fiscal stress by 1.5 standard deviations. The top left panel indicates that the shock increases the probability of sovereign default by the next quarter from 0.6% to 2.3%.[54]

The second panel shows the evolution of aggregate bank net worth and the equilibrium regions. The multiplicity region is depicted by the shaded area. Within this region, the prevalent equilibrium type is determined by household sentiments. Good sentiments (i.e. a high sunspot realization) lead to a safe equilibrium and bad sentiments result in a gambling equilibrium. The equilibrium is unique outside the multiplicity region with a safe equilibrium above it and a gambling equilibrium below. For exposition's sake, I select an initial level of net worth that lies in the multiplicity region and consider two specific scenarios. In the first scenario, sentiments come out to be good in each successive period such that there is always a safe equilibrium in the multiplicity region. In the second scenario, successive bad sentiments lead to a gambling equilibrium within the same region.

In the scenario with good sentiments, bank net worth increases rapidly and brings about an early exit from the multiplicity region. With bad sentiments, on the other hand, the economy remains "trapped" in the multiplicity region for a prolonged length of time. Since net worth is

[54]Recall that the economy immediately moves to the steady state following sovereign default. The impulse responses in Figure 2.8 correspond to a timeline where, in each successive period, it is revealed that the government remains solvent.

Figure 2.8: **Impulse responses to a sovereign risk shock**



Note: All interest rates are annualized.

retained from bank profits, the implication is that profits are lower in the gambling equilibrium compared to the safe equilibrium. This finding is surprising since, in the absence of a sovereign default event, Figure 2.8 corresponds to a timeline where a gamble on domestic sovereign bonds is successful. In other words, despite collecting a high yield from their risky bond purchases, banks make lower profits under the gambling equilibrium than the safe equilibrium.[55]

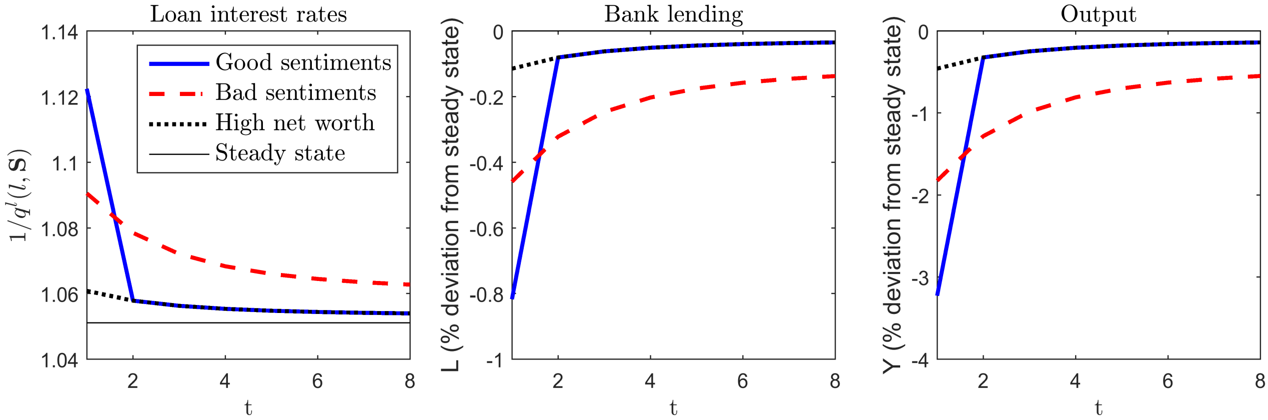
The explanation lies in the impulse responses for bank funding costs and lending. The panels in the second row of Figure 2.8 show that the gambling equilibrium entails high leverage and exposure to domestic sovereign bonds. This creates the prospect of insolvency in the case of sovereign default, which in turn increases bank funding costs to the detriment of profits.

In contrast, under the safe equilibrium, banks satisfy a solvency constraint that ensures their solvency following sovereign default. This leads to low leverage and sovereign bond exposure such that bank funding costs remain at the risk-free rate. Moreover, the solvency constraint binds in the multiplicity region such that banks reduce their lending to firms. The top right panel shows the rise in loan interest rates caused by this. Together with relatively low funding costs, the excess returns created by the rise in loan interest rates explains the rapid rise in net worth under good sentiments.

It is instructive to decompose the increase in loan interest rates, which is proportionate to

[55] Recall that banks take household sentiments as given when deciding on their optimal strategies.

Figure 2.9: **Decomposition of bank lending**



Note: All values are annualized. Bank lending and output are in percentage points.

the decline in bank lending and output. Figure 2.9 shows impulse responses for loan interest rates, aggregate bank lending and output under the same sovereign risk shock as Figure 2.8. In addition to the two scenarios above, it plots a third scenario with high initial net worth such that the safe equilibrium is unique and the solvency constraint is slack.

Compared to the (risk-free) steady state, the interest rates on loans increase and bank lending declines even in the high net worth case. This constitutes an 'efficient' decline in bank lending in view of the risk that loans become non-performing under sovereign default. In the scenario with good sentiments (and low initial net worth), bank lending initially declines significantly below the efficient level due to the deleveraging process described above, but returns back to the efficient level from the second period onwards as net worth increases. When there are bad sentiments, on the other hand, bank lending is crowded out by domestic sovereign bond purchases. This leads to a relatively mild, but still significant decline below the efficient level compared to the good sentiments case, with crowding out effects accounting for roughly 75% of the total decline in bank lending (and output). The decline is persistent, however, due to the slow increase in bank net worth.

Overall, the scenarios with good and bad sentiments highlight two alternative paths of adjustment to a sovereign risk shock when the banking sector is under-capitalized. Under the safe equilibrium, the financial soundness of the banking sector is preserved by aggressive deleveraging and there is a sharp but short-lived recession. As banks remain solvent even in the event of sovereign default, bank funding costs remain at the risk-free rate. In contrast, under bad sentiments, the economy becomes stuck in a 'gambling trap' characterized by a banking sector with high domestic sovereign bond exposure and persistent crowding out of bank lending. There is also considerable financial fragility due to the sovereign-bank nexus. If the government defaults at any point before the exit from the multiplicity region, this causes a banking crisis. As such, bank funding costs become highly correlated with sovereign bond yields.

2.4.4 Comparison with Portuguese data

This section compares the model's fit to Portuguese data. The comparative exercise is conducted by simulating the model economy under a series of sovereign risk shocks ε_d' that exactly match $q^b(\mathbf{S})^{-1}$ to quarterly Portuguese sovereign bond yields over 2010Q1-2016Q1. I also calibrate initial bank net worth to match the Tier 1 capital of the Portuguese banking sector in 2009, while the remainder of the parameters are calibrated as in Section 2.4.1.

Figure 2.10 contrasts the simulated series under good and bad sentiments (which are taken to be persistent as in the previous section) with data on the Portuguese economy. The first panel displays the sovereign default probabilities implied by the match with Portuguese sovereign bond yields. The probability of government default by the next quarter peaks at 2.78% in the final quarter of 2011.

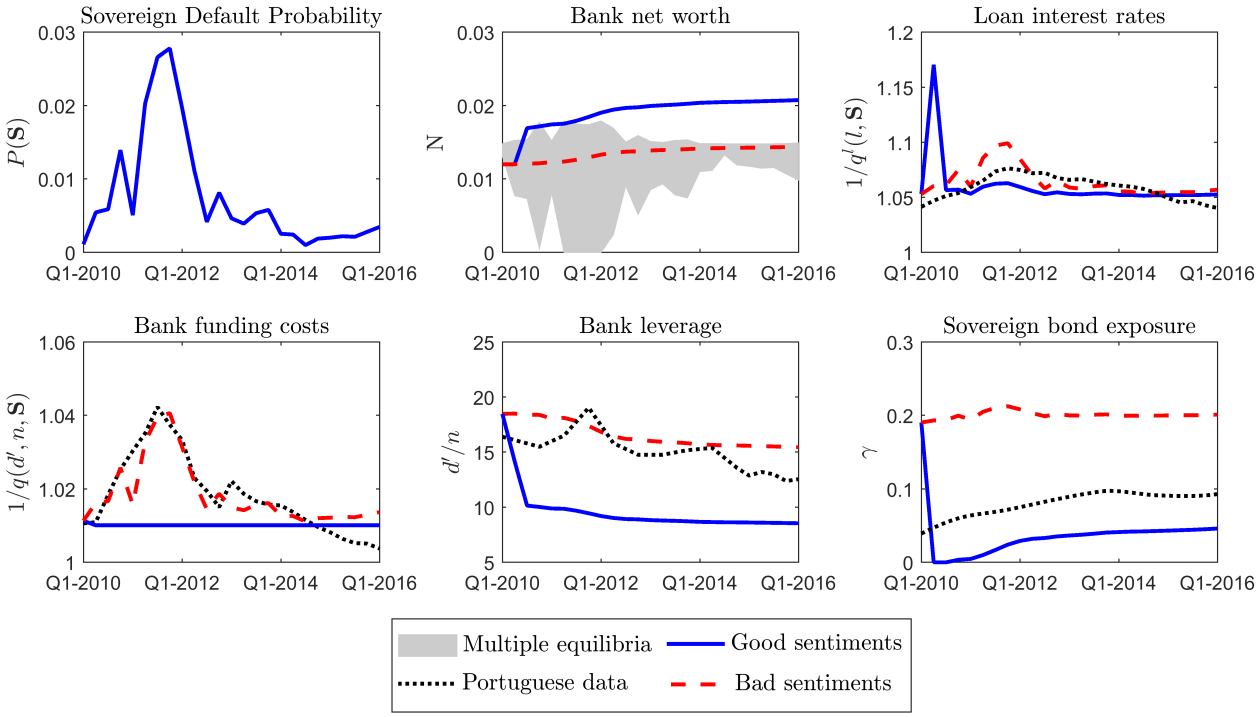
The second panel shows the simulated series for bank net worth and the multiplicity region which evolves according to changes in sovereign default probabilities as explained in Section 2.4.2. The simulation places the Portuguese economy in the region with a unique gambling equilibrium in 2010Q1, after which it enters the multiplicity region. Thereafter, bank net worth follows different paths under good and bad sentiments. As in the previous section, good sentiments result in a safe equilibrium and a rapid increase in net worth that moves the economy into the region with a unique safe equilibrium. Bad sentiments, on the other hand, lead to a gambling equilibrium with stagnating net worth such that the economy remains in the multiplicity region.

The model has partial success in emulating changes in loan interest rates. The third panel shows that the simulated series under bad sentiments captures the initial increase in loan interest rates but overshoots at the peak of the sovereign debt crisis in 2011-2012, and slightly undershoots thereafter. The simulated series under good sentiments suggests a large increase in interest rates in 2011, which is not reflected in the data. This is due to the binding solvency constraint prior to the exit from the multiplicity region, which leads to a significant decline in bank lending as in the previous section.

The model's main success is in replicating the evolution of bank funding costs. As shown in the fourth panel, the simulated series under bad sentiments provides a very close match to deposit interest rates in Portugal. Both of these series correlate highly with sovereign default probabilities. Under good sentiments, on the other hand, interest rates remain at the risk-free rate from 2010Q2 onwards. The fifth panel compares the simulated series for bank leverage with the leverage ratio of the Portuguese banking sector.[56] The simulated series under bad sentiments somewhat overshoots its counterpart in data, but captures the slow decline in bank

[56] Although the latter containts non-depository liabilities which are not directly present in the model, the nature of deposits as a choice variable captures the optimal leverage decision of banks.

Figure 2.10: **Comparison with Portuguese data**



Note: All interest rates are annualized. Sovereign default probabilities are extracted from the yield spread between Portuguese and German bonds with 3 month maturity remaining as described in section 2.4.1. The domestic sovereign bond exposure series is constructed using data from stress tests and transparency exercises conducted between 2009-2016 as well as individual bank balance sheets. See Appendix B3 for further details about data. Source: OECD, ECB, EBA.

leverage over the crisis period.

Finally, the last panel contrasts the share of funds spent on domestic sovereign bond purchases. The series under bad sentiments reflects the gradual increase in the exposure to domestic sovereign debt, but indicates a higher exposure than is observed in the data. A potential explanation for this is that the data accounts only for direct exposure via sovereign bond holdings, whereas a bank's actual exposure to domestic sovereign risk also involves indirect exposure through holdings of assets with correlated risk, such as government bonds of other risky European countries and securities issued by banks with a high exposure to these. The simulation under good sentiments indicates a large drop in exposure which is not present in the data.

Overall, the gambling equilibrium, which is consistent with bad sentiments, has more success in replicating Portuguese data than the scenario with good sentiments.

2.5 Policy analysis

In this section, I conduct a policy experiment based on an extension of the dynamic model in Section 2.3. Specifically, I extend the dynamic model to include liquidity provision (with risk transfer) as a pre-determined state variable $\bar{d}^c$.[57] For T periods, this variable follows a pre-determined path $\left\{\bar{d}_t^c\right\}_{t=0}^T$ before returning to zero permanently.[58]

I opt for this set up for two reasons. First, in the absence of debt with long-term maturity, giving banks the option to rollover their debt for T periods approximates the maturity structure of the LTROs.[59] Second, this set up allows for the solution of the extended model by iterating backwards from the end date T. This makes the additional computational burden from including the policy intervention negligible.[60]

Figure 2.11 plots the impulse responses to the same sovereign risk shock as in Section 2.4.3.[61] The first panel shows that the multiplicity region shifts upwards and expands significantly due to the policy intervention. As a result of this, the economy remains in the multiplicity region even after the deleveraging process under good sentiments. Under bad sentiments, on the other hand, net worth increases slightly faster relative to the baseline case due to the increase in gambling profits. The lower boundary of the multiplicity region also shifts up, however, and entry into the region with a unique gambling equilibrium is only narrowly avoided.

The remaining panels highlight the changes in the gambling equilibrium under the policy intervention.[62] Banks respond to liquidity provision by increasing their sovereign exposure until their funding costs return to their pre-intervention level. The top right panel shows that leverage initially increases due to the rise in borrowing by banks (both from the central bank and depositors) but falls below the baseline level over time as net worth increases more rapidly.

Overall, it appears that when liquidity provision transfers insolvency risk from depositors

[57] The changes in the deposit demand schedule and the bank's problem are similar to the two period model. I relegate the relevant expressions to Appendix B1 in the interest of brevity.

[58] The equilibrium allocation in the steady state after sovereign default is independent of $\bar{d}^c$. Therefore, there is no need to take a stance on the evolution of $\bar{d}_t^c$ following sovereign default.

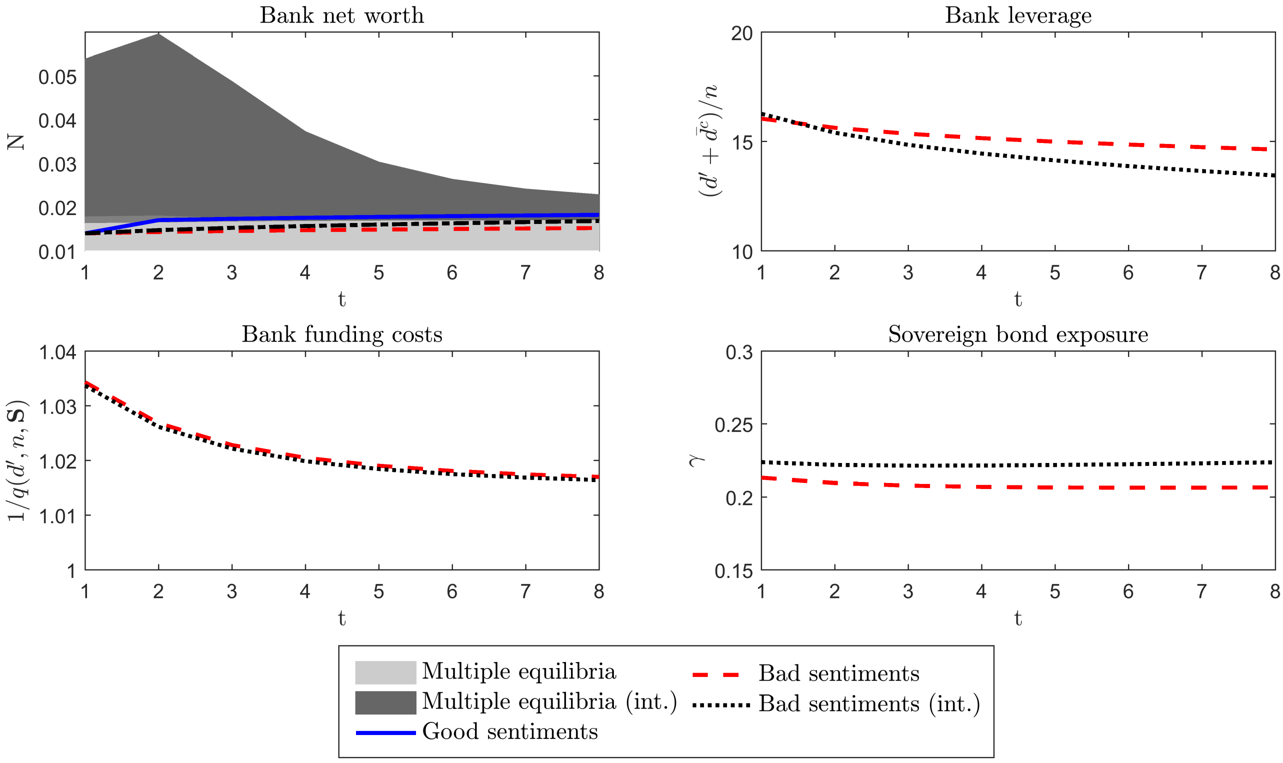
[59] The LTROs had a 3 year maturity with an early repayment option after 1 year (European Central Bank, 2011). In the context of the model, exercising the early repayment option is equivalent to choosing $d_t^c = 0$ for the remaining periods. Although this does not exactly correspond to the single window for repayment in LTROs, it emerges as a result that banks either strictly prefer to take the maximum amount of funding in each period or are indifferent to the amount of central bank liquidity they receive. Therefore, the frequency and timing of the early repayment option has no impact on the numerical results.

[60] When the policy expires at $T+1$, the extended model becomes identical to the baseline model. Therefore, future expectations at T for $\left\{\mathcal{E}_{T+1}, \Gamma_{T+1}(\mathbf{S}), X_{T+1}(\mathbf{S}), v_{T+1}^b(n, \mathbf{S})\right\}$ can be attained by taking expectations according to the solution to the baseline model. The solution to the model at period T is then attained by using the steps in section 2.3.8. Instead of iterating until convergence, the solution $\left\{\mathcal{E}_T, \Gamma_T(\mathbf{S}), X_T(\mathbf{S}), v_T^b(n, \mathbf{S})\right\}$ is used to take expectations for $T-1$. This process is repeated until $t = 0$.

[61] I calibrate $T = 12$ in line with LTROs and set $\bar{d}_t^c = \bar{d}^c < \tilde{d}^c$. The remaining parameters follow the baseline calibration in Section 2.4.1.

[62] The impulse responses under good sentiments, and those for loan interest rates are excluded as they remain identical to the baseline case in Figure 2.8.

Figure 2.11: **Liquidity provision**



Note: All interest rates are annualized.

to the central bank, it backfires not only by eliminating the safe equilibrium at low levels of net worth, but also by expanding multiplicity to higher levels of net worth such that the economy remains stuck in the gambling equilibrium for the duration of the intervention.

2.6 Conclusion

I have incorporated the framework proposed in Chapter 1 into a dynamic general equilibrium model. This has yielded an important insight: in equilibria with high bank risk-taking, the rise in funding costs in anticipation of bank default risk hinders the recovery of bank net worth. Consequently, countries with an under-capitalized banking sector may become stuck in a gambling trap characterized by bank fragility and an endogenously persistent drop in investment and output in response to negative shocks.

I have applied this model in the context of the European sovereign debt crisis, showing that its implications are consistent with the stylized facts of the crisis. I have also shown that the model can account for macroeconomic dynamics observed in Portugal in 2010-2016 in a quantitative exercise. In a policy experiment, I showed that liquidity provision similar to the European Central Bank's LTROs may backfire. These interventions strengthen incentives to gamble and may extend the length of time an economy remains in a gambling trap.

Finally, it is important to stress that the transmission mechanisms considered in this model can be interpreted in a broader context than a sovereign debt crisis. Incentives to gamble are strong whenever an asset's payoff is highly correlated to a bank's own insolvency risk. This would be the case, for example, with aggregate risky assets or illiquid assets that banks have a large pre-existing exposure to. Gambling traps may then arise when creditors are not covered by government guarantees, especially with insufficiently strict regulation to prevent excessive risk-taking. Nevertheless, these mechanisms are particularly strong during sovereign debt crises due to the triple coincidence of high correlation between sovereign default risk and aggregate risk, zero risk-weight in regulation for domestic sovereign bonds, and concerns about the credibility of government guarantees during a sovereign default episode.

Chapter 3

Shadow Banking and Market Discipline on Traditional Banks

Abstract

We present a general equilibrium banking model in which shadow banking arises endogenously and undermines market discipline on traditional commercial banks. Depositors' ability to re-optimize in response to crises imposes market discipline on traditional banks: these banks optimally commit to a safe portfolio strategy to prevent early withdrawals. With costly commitment, shadow banking emerges as an alternative banking strategy that combines high risk-taking with early liquidation in times of crisis. We bring the model to bear on the 2007-09 financial crisis in the United States, during which shadow banks experienced a sudden dry-up of funding and liquidated their assets. We derive an equilibrium in which the shadow banking sector expands to a size where its liquidation causes a fire-sale and exposes traditional banks to liquidity risk. Higher deposit rates in compensation for liquidity risk also weaken threats of early withdrawal and traditional banks pursue risky portfolios that may leave them in default. Policy interventions aimed at alleviating fire-sales fuel further expansion of shadow banking. Financial stability can be achieved with a tax on shadow bank profits or collateralized liquidity support to traditional banks.

Keywords: Shadow banking; Financial crisis; Market discipline; Fire-sales
JEL codes: E44, E58, G01, G21, G23, G28

3.1 Introduction

Recent decades have seen rapid growth in financial intermediation by non-bank entities based on a business model that combines highly-leveraged, short-term funding with risky long-term investments such as sub-prime mortgage lending. In the 2007-09 financial crisis, these "shadow banks" experienced a sudden dry-up in their funding and liquidated their assets.[63] The ensuing turmoil quickly spread to traditional commercial banks, which reduced their credit to the private sector. This has led to the deepest recession since the Great Depression, raising two important questions. First, what circumstances and mechanism lead to the emergence of shadow banking? Second, how does shadow banking affect the portfolio and funding strategies of traditional banks?

In this chapter, we propose a framework where depositors may withdraw their deposits early in reaction to crises. These "early withdrawals" constitute an optimal response to adverse changes in banks' solvency prospects and become a source of market discipline:[64] traditional banks optimally commit to a safe portfolio strategy to prevent early withdrawals. When commitment is costly, shadow banking emerges as an alternative banking strategy that combines a risky portfolio strategy with early withdrawals in times of crisis. To the best of our knowledge, this chapter presents the first model where shadow and traditional banks coexist and interact without regulatory arbitrage or direct contractual linkages.

We bring this theoretical model to bear on the 2007-09 financial crisis, its transmission to the traditional banking sector, and policy debates on shadow banking and interventions in support of banks. In doing so, we account for two key empirical facts: Shadow banks faced a sudden contraction in funding and the liquidation of their assets caused a fire-sale. Traditional banks did not suffer from withdrawals, experienced a sharp rise in their funding costs, and re-allocated their portfolios towards safe and liquid assets.

We develop our analysis by specifying a closed economy model with households, entrepreneurs, outside investors, and a banking sector. Banks collect deposits from households and choose their portfolios of safe, risky, and liquid assets; households lend to banks on terms that depend on their solvency prospects; entrepreneurs originate assets. Following news signals that revise expected asset returns, households decide whether to withdraw their deposits early and banks trade assets with outside investors in a secondary market with cash-in-the-market pricing (Allen and Gale, 1994, 2005).

A key element in the model is the equilibrium relationship between secondary market prices

[63] See Adrian and Ashcraft (2012) and Pozsar et al. (2013) for detailed descriptions of shadow banking activities.

[64] Early withdrawals are distinct from self-fulfilling bank-runs à la Diamond and Dybvig (1983). For an early withdrawal to take place, depositors must find it optimal to withdraw their funds even when no other depositor does so.

and bank strategies. During early withdrawals, shadow banks liquidate their assets in the secondary market to repay their depositors. Fire-sale externalities then create strategic substitutabilities in banks' decision to pursue a shadow banking strategy: As the shadow banking sector grows larger, its liquidation causes a deeper fire-sale, reducing the payoff from shadow banking relative to traditional banking, and bringing about an interior equilibrium where shadow and traditional banks coexist.

Analyzing the effects of fire-sales on market discipline yields important insights for vulnerability to financial crises. The key intuition is that low deposit rates strengthen early withdrawal incentives as depositors stand to lose less in terms of interest foregone. Therefore, market discipline on traditional banks is strong whenever deposit rates are low. This is precisely the case when there are no fire-sales: Interest rates on deposits are low due to the lack of liquidity risk[65] and market discipline drives traditional banks to commit to a safe portfolio strategy. With the prospect of a fire-sale, on the other hand, depositors demand higher rates in compensation for liquidity risk. This weakens early withdrawal incentives and allows traditional banks to pursue risky portfolios that may leave them in default. In equilibrium, the shadow banking sector expands to a size where its liquidation causes a fire-sale and undermines market discipline through this mechanism.

The model provides novel and important insights for policy design. We show that the outcome from policy interventions may differ substantially when adjustments in the size of the shadow banking sector are taken into account. For example, asset purchases in the secondary market are effective in alleviating fire-sales at a given shadow bank sector size. However, the ex-ante anticipation of asset purchases fuels further growth of the sector and renders the intervention completely ineffective. This troubles policymakers with time inconsistency issues: once the fire-sale is underway, they find it tempting to intervene.

Collateralized liquidity support to traditional banks also causes an expansion in shadow banking, but successfully ringfences the traditional banking sector from liquidity risk, thus restoring market discipline. Eliminating liquidity risk through deposit insurance guarantees prevents early withdrawals and necessitates the replacement of market discipline with regulatory constraints. Taxation of shadow bank profits and transfers to traditional banks are conducive to financial stability. These interventions deter banks from pursuing a shadow banking strategy and can be used to prevent the shadow banking sector from reaching a size where it causes a fire-sale.

[65] What is important here is that illiquidity leads to expected losses for depositors. We achieve this by allowing for the possibility of self-fulfilling bank-runs on illiquid banks. Another way to ensure this is to introduce idiosyncratic liquidity needs into bank assets. For example, a portion of bank assets may be backed by projects that fail without the input of additional funds.

Relationship to the literature The rapid growth of shadow banking in recent decades is well documented (see e.g. Claessens et al., 2012; Pozsar et al., 2013). This chapter builds upon a growing literature that provides micro-foundations for the existence and growth of shadow banking. Gennaioli et al. (2013) emphasize the ability of shadow banks to generate safe assets through securitization. They show that shadow banks become excessively exposed to systemic risk when low probability tail events are neglected by investors. In a similar vein, Moreira and Savov (2017) focus on liquidity transformation whereby shadow banks create money-like assets that become illiquid in times of high uncertainty. Harris et al. (2014), Plantin (2015), and Ordoñez (2017) highlight the role of regulatory arbitrage as a primary cause of shadow banking. In these studies, regulatory constraints restrict intermediation by traditional banks and create opportunities for unregulated shadow banks.

Our contribution to this literature is to show that shadow banking may arise as an equilibrium outcome without any advantages in intermediation technologies or opportunities for regulatory arbitrage. We assume, realistically, that commitment is costly and show that ex-ante identical banks endogenously cluster into traditional and shadow banking strategies, where the former optimally pay a lump-sum cost to commit to a safe portfolio strategy. In this context, commitment costs reflect any costly action undertaken by banks to resolve asymmetric information issues with their depositors, such as providing detailed balance sheet reports, eschewing opaque intermediation processes like securitization, or issuing costly equity with voting rights.

This chapter is also related to a recent strand of literature that analyses interactions between traditional and shadow banks. Gornicka (2016) considers traditional banks' incentives to gain off-balance sheet exposure by extending implicit guarantees to shadow banks. Hanson et al. (2015) present a framework where shadow and traditional banks have access to a common pool of liquidity. They show that traditional banks have a comparative advantage in holding illiquid assets with low fundamental risk when they are protected by deposit insurance guarantees. In a similar environment, Luck and Schempp (2016) find that shadow banking grows excessively large from a social viewpoint due to pecuniary externalities similar to the fire-sale externalities considered here.

We contribute to this literature by deriving an equilibrium where shadow and traditional banks coexist and interact without regulatory arbitrage or (implicit) contractual linkages. By doing this, we provide novel insights about how shadow banking affects market discipline on traditional banks. We also establish a free market benchmark to compare bank regulation and other policy interventions with.

Our modelling choices draw heavily from observations on the 2007-2009 financial crisis. Acharya et al. (2013) document the collapse in the market for asset-backed commercial papers at the onset of the crisis while Gorton and Metrick (2012) and Krishnamurty et al. (2014)

show a similar contraction in repo markets. Together, these two markets accounted for the vast majority of funding for shadow banks. Covitz et al. (2013) find that the dry-up in funding for shadow banks was associated with a rise in macro-financial risks such as uncertainty about sub-prime mortgages values. In this chapter, the definitive characteristic of the shadow banking strategy is its vulnerability to early withdrawals which closely resemble these events. Consistent with Covitz et al. (2013), early withdrawals are triggered by a negative revision in expected asset payoffs.

We propose that fire-sales caused by the liquidation of shadow banks play a key role in the spread of financial instability to traditional banks. Shleifer and Vishny (2011) provide an extensive review of the literature on fire-sales while Krishnamurthy (2010), Merrill et al. (2012), and Mitchell and Pulvino (2012) provide empirical evidence for fire-sales during the financial crisis. Our mechanism is similar to Diamond and Rajan (2005) in that (shadow) bank failures cause contagion by aggravating liquidity shortages in the rest of the financial sector.

Finally, like Diamond and Rajan (2011), we find that illiquidity leads to greater risk-taking by traditional banks. In Diamond and Rajan (2011), this is a consequence of risk-shifting whereas our finding arises due to interactions between deposit rates and market discipline that stem from threats of early withdrawal by depositors. Crucially, our mechanism can generate solvency risk[66] where none previously existed, whereas the risk-shifting mechanism exacerbates pre-existing risks.

Layout We proceed as follows: Section 2.2 presents motivating evidence from the 2007-09 financial crisis. Section 3.3 describes the core mechanisms of the model in a simple framework without liquidity risk. Section 3.4 presents the complete model with liquidity risk. Section 3.5 provides the numerical results. Section 3.6 conducts policy analysis. Section 3.7 concludes.

3.2 Motivating evidence

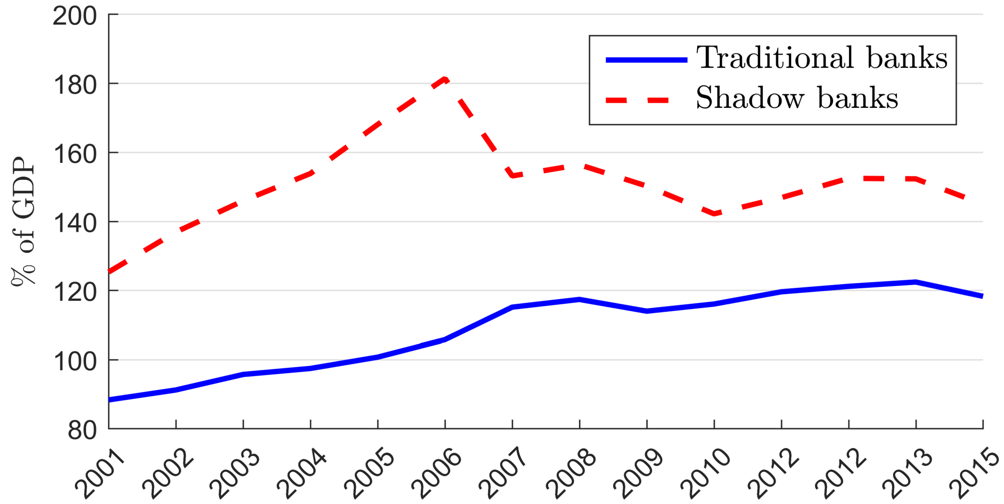
In this section, we present four key stylized facts about shadow banking and the 2007-09 financial crisis in the United States.

Fact 1. *The shadow banking sector expanded rapidly until its collapse in 2007. The traditional banking sector grew at a slower rate but did not suffer from a collapse.*

Figure 3.1 shows that shadow bank assets expanded from 125% of GDP in 2002 to over 180% at its peak in 2007. This rapid growth came to an end with a collapse during the financial

[66] We distinguish between "solvency risk" which is the risk of fundamental insolvency after holding assets to maturity, and "liquidity risk" which refers to the prospect of bankruptcy due to withdrawals before assets reach maturity.

Figure 3.1: **Shadow and traditional bank assets**



Note: Financial assets when available, otherwise total assets. Traditional banks refer to all deposit-taking corporations. Shadow banks refer to all financial corporations that are not classified as central banks, banks, insurance corporations, pension funds, public financial institutions, or financial auxiliaries.

Source: Financial Stability Board (2017) and the World Bank

crisis. In 2015, shadow bank assets amounted to 145% of GDP. The traditional banking sector also expanded but at a relatively modest pace with its assets increasing from 88% of GDP in 2002 to 118% in 2015. Although there was no contraction in traditional banking during the crisis, the shadow banking sector remained larger throughout the period 2002-2015.

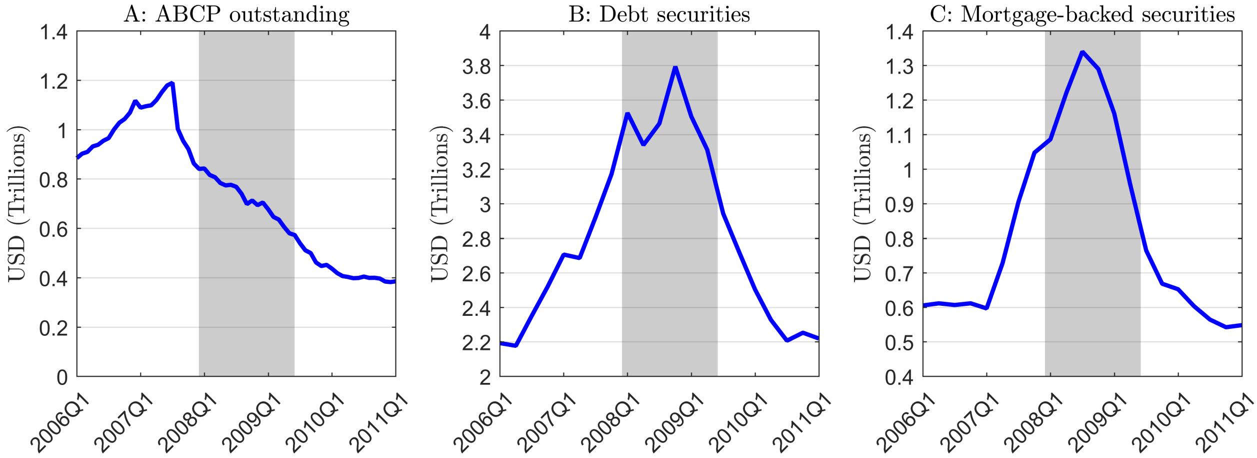
In our model, the size of the shadow banking sector is endogenously determined through free entry and may rise for two reasons: an increase in the liquidity available in secondary markets and a rise in the cost of commitment. Recent financial innovations such as securitization may have increased both the commitment cost and the thickness of secondary markets. In Section 3.6, we also analyze the role of policy interventions in the expansion of shadow banking. We show that expectations that policymakers will lean against a fire-sale increase the equilibrium size of the shadow banking sector in the same manner as the deepening of secondary markets.[67]

Fact 2. *At the onset of the financial crisis, shadow banks experienced a sudden dry-up of funding and liquidated their assets.*

At their peak of $1.2 trillion in July 2007, asset-backed commercial papers (ABCP) were the largest money market instrument in the United States and constituted the main source of funding for shadow banks. Following rising mortgage default rates and the suspension of withdrawals by a number of funds, the market for ABCP contracted by $350 billion in the

[67] This may be implemented directly through asset purchases or indirectly by bailing out financial institutions which would have contributed to the excess supply of assets.

Figure 3.2: **ABCP markets and shadow bank assets**



Note: Shaded areas indicate US recessions. Shadow banks refer to money market mutual funds, security brokers and dealers, and issuers of asset-backed securities. Mortgage-backed securities refer to agency- and GSE-backed securities.

Source: Financial Accounts of the United States

second half of 2007 and a further \$400 billion by the end of 2009 (see Panel A of Figure 3.2). Faced with this sudden contraction in funding, shadow banks liquidated their asset holdings. Panels B and C show that shadow banks sold \$1.5 trillions of debt securities between 2008Q3 and 2010Q1, approximately half of which were mortgage-backed securities.[68]

In our model,shadow banks are vulnerable to early withdrawals which closely resemble these events. Early withdrawals take place following an adverse change in macro-financial fundamentals: a bad news signal that leads to a downward revision in expected asset returns. These withdrawals constitute an optimal response by creditors to bad news rather than self-fulfilling bank-runs in the fashion of Diamond and Dybvig (1983).

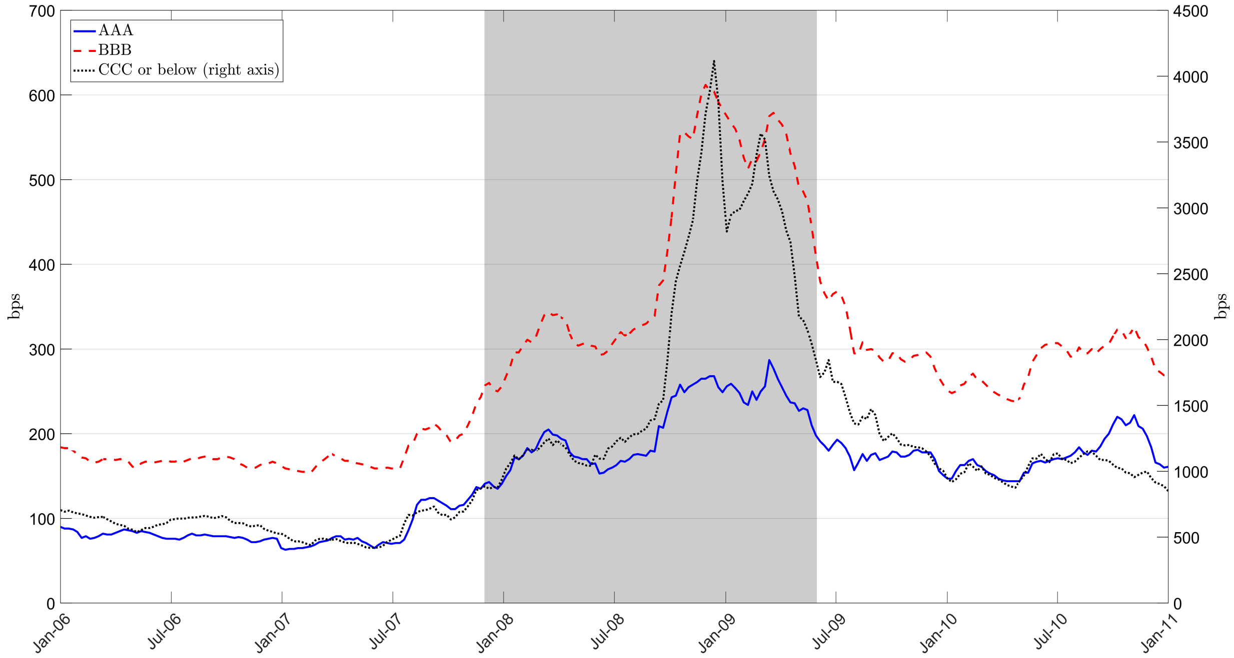
During a withdrawal, shadow banks are forced to liquidate their assets in the secondary market. To the extent that this generates an adjustment in secondary market prices, there are consequences for traditional banks even in the absence of implicit guarantees or any other form of direct exposure between the traditional and shadow banking sectors.

Fact 3. *Spreads between private debt securities and Treasury bonds increased sharply during the crisis. Securities with higher perceived risk experienced greater increases in spreads.*

Figure 3.3 shows the evolution of yield spreads between corporate bonds (grouped according to their credit rating) and Treasury bonds of comparable maturity. It is notable that spreads

[68]It is notable that shadow banks' holdings of debt securities peaked in 2008Q3 nearly a year after the contraction in ABCP. One explanation is that shadow banks nearing bankruptcy prioritized the sale of liquid assets while increasing their exposure to higher yield mortgage-backed securities in a gamble for resurrection. See e.g. Ari (2017) for a model where banks facing adverse funding conditions and high aggregate risk find it optimal to behave in this manner.

Figure 3.3: **Spreads on corporate bonds**



Note: Shaded areas indicate US recessions. AAA (BBB) refers to the spread between Moody's Seasoned Aaa (Baa) Corporate Bond and Treasury bonds with 10-year maturity (constant). CCC or below refers to the option-adjusted spread between the Bank of America Merrill Lynch US Corporate C Index and a spot Treasury curve.

Source: Moody's Investor Services, Bank of America Merrill Lynch

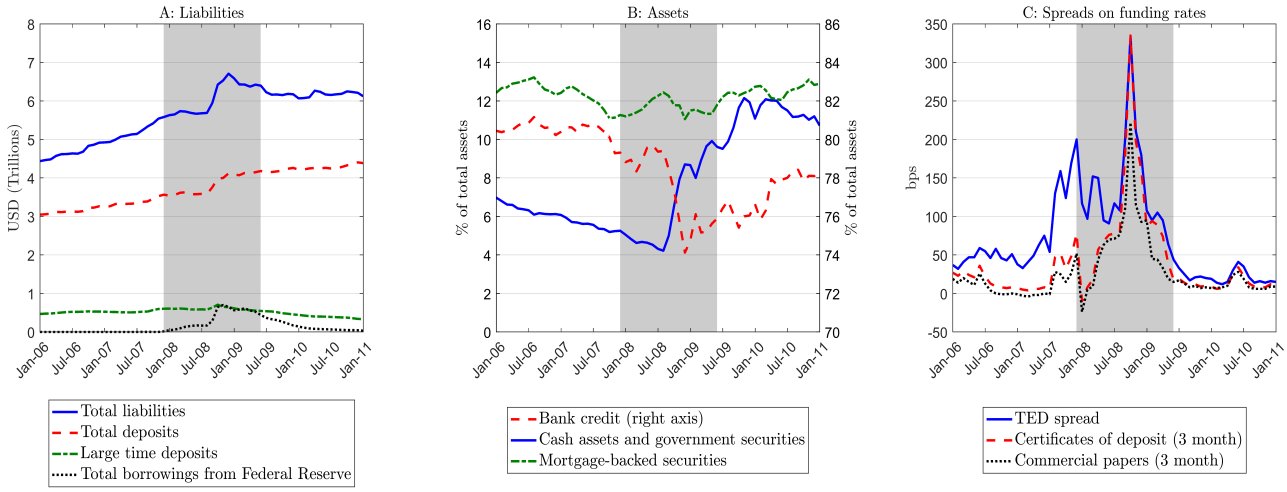
peaked in the last quarter of 2008, coinciding with the liquidation of shadow bank assets. Although there was a sharp rise in spreads for every rating category, the increase was greater for lower-rated corporate bonds.

Our model generates changes in secondary market prices that are consistent with these movements in spreads. When there is cash-in-the-market pricing, the liquidation of shadow bank portfolios causes a fire-sale and (illiquid) assets trade at a significant discount. Portfolio re-allocation by traditional banks from risky to safe and liquid assets also contributes to the fire-sale. We provide evidence for and further discuss portfolio re-allocation under Fact 4.

Fact 4. *Traditional banks re-allocated their portfolios toward safe and liquid assets and were able to increase their liabilities. At the same time, they faced a rise in their funding costs.*

Panel A of Figure 3.4 shows that traditional banks increased their liabilities during the crisis. Although the increase in liabilities was driven by Federal Reserve funding and a rise in deposits protected by deposit insurance guarantees, there was also no decline in large time deposits. Since large time deposits are neither checkable, nor insured, the implication is that traditional banks were perceived to be safe enough to preclude withdrawals even in the absence of government guarantees.

Figure 3.4: **Traditional banks during the crisis**



Note: Shaded areas indicate US recessions. Traditional banks refers to "large domestically chartered commercial banks" which are defined as the top 25 domestically chartered commercial banks, ranked by domestic assets. Cash assets and govenment securities include vaullt cash, cash items in process of collection, balances due from depository institutions and Federal Reserve Banks, and liabilities of the U.S. government, U.S. government agencies and U.S. government-sponsored enterprises. The TED spread is calculated as the spread between LIBOR based on US dollars and Treasury Bills. Certificate of deposit and commercial paper spreads are between the secondary market rates of these securities and the Effective Federal Funds Rate. All spreads refer to 3 month maturities.

Source: Board of Governors of the Federal Reserve System, the Federal Reserve Bank of St. Louis

Changes in traditional bank portfolios shown in Panel B lend further support to this interpretation. During the crisis, traditional banks re-allocated their portfolios from bank credit (which is risky and illiquid) to cash assets and government securities (which are liquid and safe). Mortgage-backed securities that were at the epicenter of the financial crisis accounted for only 12% of total assets and this did not change significantly over the crisis.

It is important to note, however, that while traditional banks did not experience a dry-up in funding, there was a significant increase in their funding costs. Panel C shows that the TED spread (for interbank loans) increased to 200 basis points in December 2007 and 335 basis points in October 2008. The spreads on certificates of deposits and commercial papers also increased significantly.

In the following sections, we show that the interaction between market discipline and liquidity risk may account for these observations. In our framework, market discipline stems from the threat of early withdrawal by depositors. To avoid an early withdrawal, traditional banks commit to a minimum recovery rate on deposits which we refer to as a "no-withdrawal constraint" and respond to bad news by re-allocating their portfolios towards safe assets.

The fire-sale discussed under Fact 3 interacts with this mechanism through two distinct transmission channels. First, since risky assets are discounted to a greater extent than safe assets, it reduces traditional banks' capacity for portfolio re-allocation. To satisfy the no-withdrawal constraint, traditional banks are then forced to reduce their risky asset holdings ex-ante. This creates an excess return that increases the expected payoff associated with traditional banking. As the extent of the fire-sale is proportionate to the size of the shadow banking sector, equilibrium is achieved through this mechanism. In the next section, we analyze this mechanism and the properties of the equilibrium in a simple model without liquidity risk.

Second, the fire-sale leaves traditional banks illiquid. This drives a wedge between solvency and liquidity, leaving traditional banks vulnerable to self-fulfilling bank-runs as per Diamond and Dybvig (1983). In Sections 3.4 and 3.5, we extend the simple model to allow for liquidity risk (i.e. the possibility of bank-runs) and show that this undermines market discipline on traditional banks: Depositors demand higher interest rates to compensate for liquidity (bank-run) risk, generating a rise in deposit rates similar to Panel C. Higher deposit rates in turn weaken the threat of early withdrawal, bringing about a relaxation in the no-withdrawal constraint and greater risk-taking by traditional banks. In equilibrium, the shadow banking sector causes financial instability through this mechanism.

3.3 A simple model

We consider a financial economy populated by four agents: households, banks, entrepreneurs, and outside investors. Events unfold over three time periods (see Figure 3.5 for a graphical timeline). In the first period, banks collect deposits from households and invest in (safe and liquid) cash and (risky and illiquid) assets originated by entrepreneurs.[69]

The second period begins with a public news signal that leads to a revision of expected asset returns. With probability q, the signal harbors "bad news" leading to a decline in expected asset returns. After observing the signal, households decide whether to withdraw their deposits early and banks trade assets with outside investors in a secondary market.

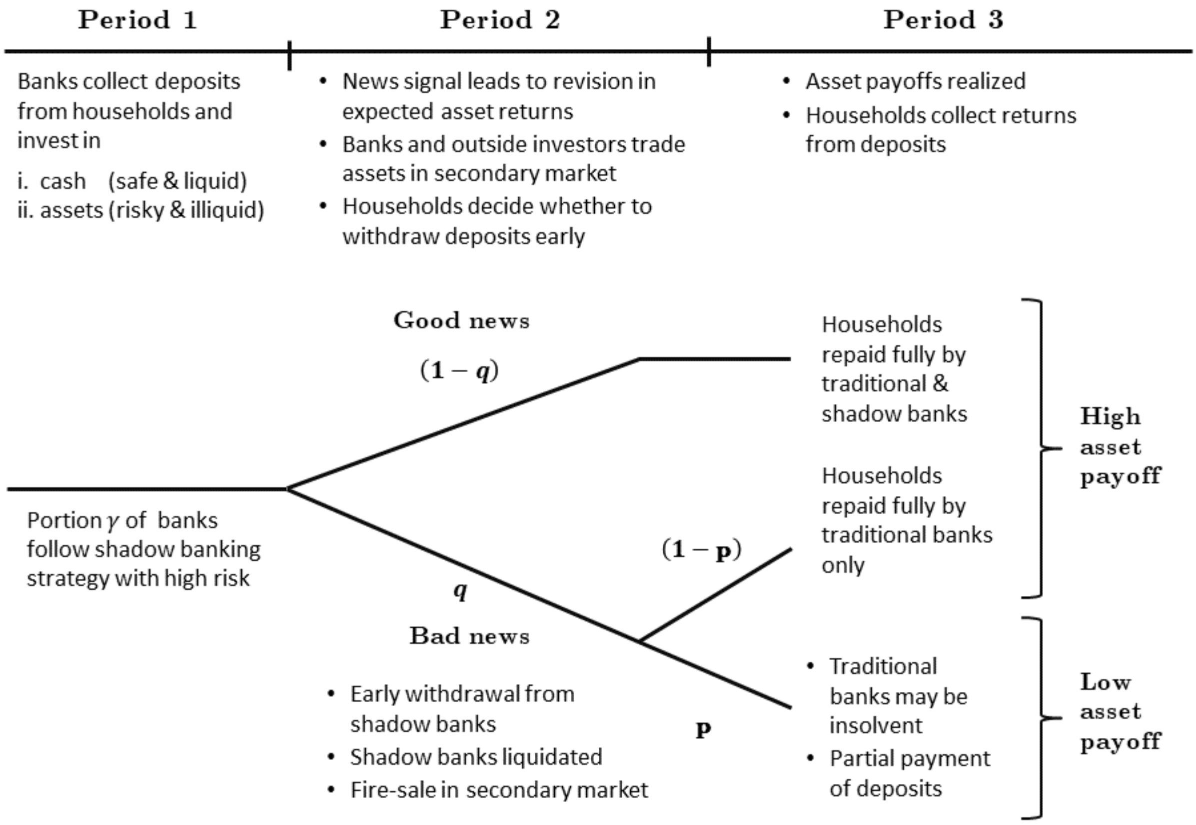
In the third period, assets yield a high or low payoff contingent on economic fundamentals. Following bad news in the second period, fundamentals may turn out to be weak with (conditional) probability p, leading to a low payoff from assets. Depending on their investment strategy, banks may then be left with insufficient funds to pay the promised return to their depositors. In this case, they become insolvent under limited liability and a haircut proportionate to their funding shortfall is imposed on deposits.[70]

The key friction in the model is costly commitment. Specifically, we assume that banks cannot credibly commit to a safe investment strategy unless they pay a commitment cost

[69] In Section 3.4, we also introduce a safe but illiquid asset.

[70] Fundamentals turn out to be strong with certainty after good news (which takes place with probability $(1-q)$) and with (conditional) probability $(1-p)$ after bad news. In this case, assets yield a high payoff and banks are solvent.

Figure 3.5: **Timeline**



$\tau > 0$.[71] Due to this friction, ex-ante identical banks optimally cluster into two distinct groups according to their investment strategies. An endogenous share $\gamma \in [0, 1]$ of banks do not pay the commitment cost and follow a "shadow banking" strategy that entails high risk-taking. Following bad news, households optimally withdraw their deposits from shadow banks due to concerns about their solvency prospects. Shadow bank assets are then liquidated in the secondary market at an endogenous fire-sale discount and depositors receive the liquidation value. The remaining banks pay the cost τ to credibly commit to a portfolio that is safe enough to prevent an early withdrawal. We refer to this as a "traditional banking" strategy.

It is important to note that early withdrawals are distinct from self-fulfilling (Diamond-Dybvig) bank-runs, as they are an optimal response to a change in banks' solvency prospects rather than a consequence of strategic complementarities across households. In other words, an early withdrawal takes place when households find it optimal to withdraw their deposits

[71] We do not explicitly model the underlying principal-agent problem between banks and their depositors but provide two motivating examples. First, bank balance sheets may not be fully transparent. The commitment cost would then reflect any costly action taken by banks to increase transparency such as abiding by reporting requirements or foregoing opaque intermediation processes such as securitization. Second, limited enforcement of contracts may allows banks to revise their investment strategies after collecting deposits. The commitment cost could then be in the form of issuing costly equity with voting rights.

regardless of the extent of withdrawals by other households.

Before explaining these activities in detail, we briefly describe some notational conventions. Table 3.1 provides a list of variables and parameters. We denote variables that pertain to a shadow (traditional) banking strategy with a superscript 'SB' ('TB'). Deposits, cash and (risky) assets are respectively labelled as (D, M, I). To distinguish between deposits per bank and per household, we denote the latter as d in lower case. The effective return from deposits is contingent on solvency and the time of withdrawal. Upon maturity in the third period, deposits pay an interest rate R when the bank is solvent and a recovery rate V otherwise. When they are withdrawn early, deposits pay a liquidation value θ. Asset prices in the first and second period are labelled as (P_1, P_2). In period 3, assets yield a payoff σ_h when fundamentals are strong and σ_l otherwise. We simplify notation by normalizing the risk-free rate to $R^* = 1$ and the unconditional expectation of asset payoffs to unity such that[72]

$$(1 - qp)\,\sigma_h + qp\sigma_l = 1 \tag{3.1}$$

3.3.1 Agents and their optimal strategies

3.3.1.1 Entrepreneurs

In the first period, each bank has access to a separate but ex-ante identical island of entrepreneurs which use capital K to produce assets I_1 with a Cobb-Douglas production technology

$$I_1 = AK^{\alpha} \tag{3.2}$$

where $A > 0$ is a productivity parameter and $\alpha \in (0, 1)$ is the standard Cobb-Douglas elasticity. The representative entrepreneur's problem can be written as

$$\max_{I_1, K} \; P_1 I_1 - K$$

subject to the production technology (3.2) where P_1 is the asset price.[73] This yields a first order condition that can be interpreted as an upward-sloping asset supply schedule

$$P_1 = \frac{1}{\alpha A^{\frac{1}{\alpha}}} I_1^{\frac{1-\alpha}{\alpha}} \tag{3.3}$$

Due to relationship lending frictions, banks may only purchase assets from entrepreneurs

[72] Note that assets may still have a return above unity since (P_1, P_2) are endogenous.

[73] We normalize the rental rate of capital to unity.

Table 3.1: **Notation**

Variables

Label	Description
D	Deposits (per bank)
d	Deposits (per household)
I	Assets
M	Cash
R	Interest rate on deposits
K	Capital
P_1, P_2	Asset prices
$\widetilde{I}$	Excess supply in secondary market
$\widetilde{K}$	Outside project
ϕ	Fire-sale discount
θ	Liquidation value
V	Recovery rate
Π	Bank profits
γ	Share of shadow banks
C	Consumption

Parameters

Label	Description
q	Prob. of bad news
p	Prob. of weak fundamentals
σ_h, σ_l	Asset payoffs
A	Productivity
α	Cobb-Douglas elasticity
R^*	Risk-free rate
E	Household endowment
$\widetilde{E}$	Outside investor endowment
τ	Commitment cost

in their own island.[74,75] This also constitutes a barrier to entry that gives banks market power over entrepreneurs, allowing them to extract a mark-up

$$\frac{\partial P_1}{\partial I_1}\frac{I_1}{P_1} = \mu > 0$$

In the interest of tractability, we do not explicitly model the bargaining process between banks and entrepreneurs but restrict the mark-up to

$$\mu \leq \bar{\mu} \equiv \frac{(1-q)(\sigma_h - 1)}{1-(1-q)\sigma_h} \tag{3.4}$$

This restriction only serves to simplify the bank's problem in Section 3.3.1.4 and we show in Appendix C4.2 that it can be relaxed without altering the properties of the equilibrium solution.

[74]Implicitly, we assume that entrepreneurs may costlessly produce a pseudo-asset that pays zero return and banks may only monitor entrepreneurs in their own island. The same friction also bars households from investing directly in assets.

[75]We also assume that banks may only trade assets in the second period. Therefore, the asset price P_1 is specific to each bank and is best interpreted as the cost of origination rather than the price of a tradable asset.

3.3.1.2 Secondary market and outside investors

In the second period, banks and outside investors trade assets in a secondary market. First, we consider the case after good news. Since assets yield a high payoff σ_h with certainty after good news, there are no early withdrawals. Secondary markets then clear without a fire-sale at a price σ_h. Whilst trade may take place, the pricing of assets at their expected payoff ensures that it is completely inconsequential to the equilibrium allocation. Therefore, we abuse notation slightly by using the subscript 2 to denote variables after bad news throughout the chapter.

Following bad news, the liquidation of shadow banks and portfolio re-allocation by traditional banks leads to an excess supply of assets

$$\widetilde{I} = \gamma I_1^{SB} + (1-\gamma)\left(I_1^{TB} - I_2^{TB}\right) > 0$$

where γ is the share of shadow banks within the banking sector, I_1^{SB} is shadow bank assets, and the second term represents the net sale of assets by traditional banks. Outside investors with a lower valuation for assets then become the marginal buyers and the secondary market price is given by

$$P_2 = \phi\left[(1-p)\,\sigma_h + p\sigma_l\right]$$

where the fire-sale discount $\phi \in [0,1]$ adjusts to clear markets and the term in brackets is the expected payoff conditional on bad news.

Our set up for outside investors is based on Stein (2012). Outside investors begin the second period with an endowment $\widetilde{E}$ and allocate their funds between asset purchases in the secondary market and an outside project $\widetilde{K}$ that yields a safe payoff $g\left(\widetilde{K}\right)$ where $g'(.) > 0$, $g''(.) < 0$. Their first order condition equates the expected return between the two investment opportunities and yields an implicit expression for the market-clearing fire-sale discount.

$$\phi = \frac{1}{g'\left(\widetilde{E} - \phi\left[(1-p)\,\sigma_h + p\sigma_l\right]\widetilde{I}\right)} \tag{3.5}$$

Without loss of generality, we can write this as $\phi = f\left(\widetilde{I}\right)$ where $f(.)$ is a continuous and decreasing function.

We restrict the fire-sale discount to values within the range $\phi \in \left(\underline{\phi}, \overline{\phi}\right]$ by imposing the following restrictions on $f(.)$ (and hence on $\widetilde{E}$ and $g(.)$)

$$\underline{\phi} < f\left((A\alpha^\alpha)^{\frac{1}{1-\alpha}}\left((1-q)\,\frac{\sigma_h}{1+\mu} + q\sigma_l\right)^{\frac{\alpha}{1-\alpha}}\right) < \overline{\phi} \tag{3.6}$$

where

$$\begin{aligned}\underline{\phi} &\equiv \frac{\sigma_l}{(1-p)\,\sigma_h + p\sigma_l} \geq 0 \\ \bar{\phi} &\equiv \min\left[1, \frac{(1-q)\,\frac{\sigma_h}{1+\mu} + q\sigma_l}{(1-p)\,\sigma_h + p\sigma_l}\right] > \underline{\phi}\end{aligned}$$

The upper bound of this restriction ensures that the fire-sale deepens as the shadow banking sector grows, and is necessary for bringing about an interior equilibrium where shadow and traditional banks coexist.[76] The lower bound, on the other hand, only serves to simplify our exposition by eliminating the possibility of an (interior) equilibrium case with slightly different properties to the one we describe in Section 3.3.2. We discuss the consequences of relaxing the lower bound restriction in Appendix C5.

3.3.1.3 Households

There is a unit continuum of risk neutral households which derive utility only from final period consumption, making their utility maximization problem equivalent to maximizing expected consumption[77]

$$E[C] = (1-q)\,C_{gh} + q\,(1-p)\,C_{bh} + qpC_{bl} \tag{3.7}$$

where (C_{gh}, C_{bh}, C_{bl}) are respectively consumption under good news and bad news with high and low payoff from assets. In the first period, households allocate their endowment E between deposits in traditional and shadow banks $\left(d^{TB}, d^{SB}\right)$ and cash M_1 which transfers funds to the next period at zero net return. The first period budget constraint is

$$d^{SB} + d^{TB} + M_1 = E \tag{3.8}$$

When the second period begins with good news, there are no withdrawals and households simply retain their cash holdings M_1. In the third period, consumption is then given by

$$C_{gh} = M_1 + d^{TB}R^{TB} + d^{SB}R^{SB} \tag{3.9}$$

where $\left(R^{TB}, R^{SB}\right)$ represent the deposit interest rates at traditional and shadow banks.

Following bad news, on the other hand, households observe the decline in expected asset returns and decide whether to withdraw their deposits early. Before we consider this decision, we solve the household's optimization problem in the first period taking it as given that only

[76] See Section 3.3.2 for a formal definition.

[77] These assumptions only serve to simplify the exposition. Our findings remain the same under risk aversion and period-by-period discounting.

shadow banks face an early withdrawal. The second period budget constraint under bad news can then be written as

$$M_2 = M_1 + \theta^{SB} d^{SB} \tag{3.10}$$

where θ^{SB} is the liquidation value of shadow bank portfolios. In the third period, traditional banks may be rendered insolvent by a low payoff from assets. Therefore, consumption is contingent on asset payoffs

$$\begin{aligned} C_{bh} &= M_2 + d^{TB} R^{TB} & (3.11) \\ C_{bl} &= M_2 + V d^{TB} R^{TB} & (3.12) \end{aligned}$$

with V representing the recovery rate of traditional bank deposits.[78]

The representative household chooses $\left\{d^{TB}, d^{SB}, M_1, M_2\right\}$ to maximize its expected consumption (3.7) subject to (3.8)-(3.12). The first order conditions yield the following expressions for interest rates

$$\begin{aligned} R^{TB} &= 1 + \frac{qp\,(1-V)}{1-qp\,(1-V)} & (3.13) \\ R^{SB} &= 1 + \frac{q}{1-q}\left(1-\theta^{SB}\right) & (3.14) \end{aligned}$$

Observe that the interest rate on traditional bank deposits decreases with a rise in the recovery rate V, while that of shadow banks decreases with a higher liquidation value θ^{SB}. Traditional banks may borrow at the risk-free rate $R^{TB} = 1$ when they guarantee a complete repayment of deposits $V = 1$ while shadow banks must be completely liquid with $\theta^{SB} = 1$.

Early withdrawal decision and market discipline

Following bad news, it is optimal for households to withdraw their deposits when doing so increases their expected consumption $(1-p)\,c_{bh} + pc_{bl}$ conditional on bad news. Proposition 3.1 describes the outcome of the early withdrawal decision.

Proposition 3.1 *For all $\phi > \underline{\phi}$, it is optimal for households to withdraw their deposits from shadow banks after bad news. Traditional banks may avoid an early withdrawal by committing to a minimum recovery rate*

$$V \geq \bar{V} \equiv \frac{1}{p}\left(\frac{1}{R^{TB}} - (1-p)\right) \tag{3.15}$$

[78] The liquidation value θ^{SB} and recovery rate V are endogenous and depend on the investment strategy of traditional and shadow banks. We elaborate further on this in Section 3.3.1.4.

In equilibrium, the recovery rate and interest rates on traditional bank deposits are given by

$$\bar{V} = R^{TB} = 1 \tag{3.16}$$

Proof. Provided in Appendix C1 ■

Note that (3.15) establishes a no-withdrawal constraint for traditional banks. This constraint is the key difference between shadow and traditional banking strategies, and generates similar behaviour to the observations discussed under Fact 4. To satisfy this constraint, traditional banks react to bad news by re-allocating their portfolio from risky to safe assets.

The proposition also shows that the no-withdrawal constraint imposes market discipline on traditional banks, eliminating insolvency risk and reducing their funding costs to the risk-free rate. This is the outcome of a virtuous cycle between deposit rates R^{TB} and the minimum recovery rate $\bar{V}$. As traditional banks guarantee a minimum recovery rate, households demand less compensation for insolvency risk and deposit rates decrease as per (3.13). Lower deposit rates in turn strengthen the threat of early withdrawal as households stand to lose less in terms of interest foregone. Therefore, the minimum recovery rate $\bar{V}$ increases until all solvency risk is eliminated in equilibrium.

It is important to note that this result is contingent on the lack of liquidity risk. In Section 3.4, we show that even a small risk of a (Diamond-Dybvig) bank run on traditional banks reverses this virtuous cycle, leading to a rise in insolvency risk and funding costs.

3.3.1.4 Banks

There is a unit continuum of ex-ante identical, risk neutral banks. In the first period, banks collect deposits D from households and purchase assets I_1 from entrepreneurs at price P_1 as well as holding cash M_1. Their first period budget constraint can then be written as

$$P_1 I_1 + M_1 = D \tag{3.17}$$

In the second period, banks trade assets in the secondary market. As discussed in Section 3.3.1.2, following good news, assets are priced at their expected payoff in the secondary market and trade is inconsequential. Bank profits (in the third period) are given by

$$\Pi_{gh} = \sigma_h I_1 + M_1 - DR$$

Following bad news, banks that are subject to an early withdrawal have a liquidation value

$$\theta = \min\left\{1, \frac{P_2 I_2 + M_2}{D}\right\} \tag{3.18}$$

and those that are not face the second period budget constraint

$$P_2 I_2 + M_2 = P_2 I_1 + M_1 \tag{3.19}$$

Observe that a fire-sale reduces the liquidation value as well as the maximum cash M_2 banks may attain through portfolio re-allocation. When assets yield a high payoff σ_h in the third period, banks make a profit

$$\Pi_{bh} = \sigma_h I_2 + M_2 - DR$$

while limited liability binds under a low asset payoff σ_l.[79] Under limited liability, banks make zero profits and deposits pay a recovery rate

$$V = \min\left\{1, \frac{\sigma_l I_2 + M_2}{DR}\right\} \tag{3.20}$$

which is proportional to the shortfall of funds.

The commitment cost $\tau > 0$ creates a discontinuity in the optimization problem of banks such that it can be evaluated as a choice between two distinct investment strategies. Under a shadow banking strategy (labelled as 'SB'), banks do not pay the commitment cost and anticipate an early withdrawal after bad news. Under a traditional banking strategy (labelled as 'TB'), banks pay the cost τ and commit to satisfying the no-withdrawal constraint (3.15). We find it convenient to to write the no-withdrawal constraint in terms of the choice variables by combining (3.15) and (3.20).[80]

$$\sigma_l I_2^{TB} + M_2^{TB} \geq \bar{V} D^{TB} R^{TB} \tag{3.21}$$

In the first period, banks adopt the strategy that leads to the highest expected payoff such that shadow banking is preferred when

$$E\left[\Pi^{SB}\right] \geq E\left[\Pi^{TB}\right] - \tau \tag{3.22}$$

where $E\left[\Pi^{SB}\right]$ and $E\left[\Pi^{TB}\right]$ are the expected payoffs associated with shadow and traditional banking. Below, we solve the optimization problem under each strategy and attain an expression for expected payoffs by combining banks' first order conditions with those of households from Section 3.3.1.3. In doing so, we take the secondary market price P_2 as given.

[79] See Appendix C2 for the relevant proof.

[80] To simplify the exposition, we treat the commitment cost τ as a utility cost, thereby omitting it from the no-withdrawal and budget constraints. Since τ is small relative to total bank assets, its inclusion would have a negligible impact on the equilibrium outcome.

Shadow banking

Shadow banks choose $\left\{I_1^{SB}, M_1^{SB}, D^{SB}\right\}$ to maximize their expected profits

$$E\left[\Pi^{SB}\right] = (1-q)\left(\sigma_h I_1^{SB} + M_1^{SB} - D^{SB}R^{SB}\right) \tag{3.23}$$

subject to the budget constraint (3.17). Since limited liability binds after an early withdrawal, shadow banks only internalize the payoff after good news. Lemma 3.1 provides the solution the shadow bank's problem. It shows that, even without a fire-sale, shadow bank deposits are not paid in full during an early withdrawal ($\theta^{SB} < 1$) and funding costs of shadow banks are above the risk free rate ($R^{SB} > 1$) as a consequence.

Lemma 3.1 *Combining the solution to the shadow bank's problem given by (C51) and (C52) with the household first order condition (3.14) yields the following expressions for* $\left(\theta^{SB}, R^{SB}, E\left[\Pi^{SB}\right], I_1^{SB}\right)$

$$\begin{aligned}
\theta^{SB} &= \frac{(1+\mu)P_2}{(1-q)\sigma_h + q(1+\mu)P_2} < 1 \ \forall \ \phi \in [0,1] \\
R^{SB} &= \frac{\sigma_h}{(1-q)\sigma_h + q(1+\mu)P_2} > 1 \ \forall \ \phi \in [0,1] \\
E\left[\Pi^{SB}\right] &= (1-q)\frac{\mu}{1+\mu}\sigma_h I_1^{SB} \\
I_1^{SB} &= (A\alpha^\alpha)^{\frac{1}{1-\alpha}}\left((1-q)\frac{\sigma_h}{1+\mu} + qP_2\right)^{\frac{\alpha}{1-\alpha}}
\end{aligned}$$

Proof. Provided in Appendix C3 ■

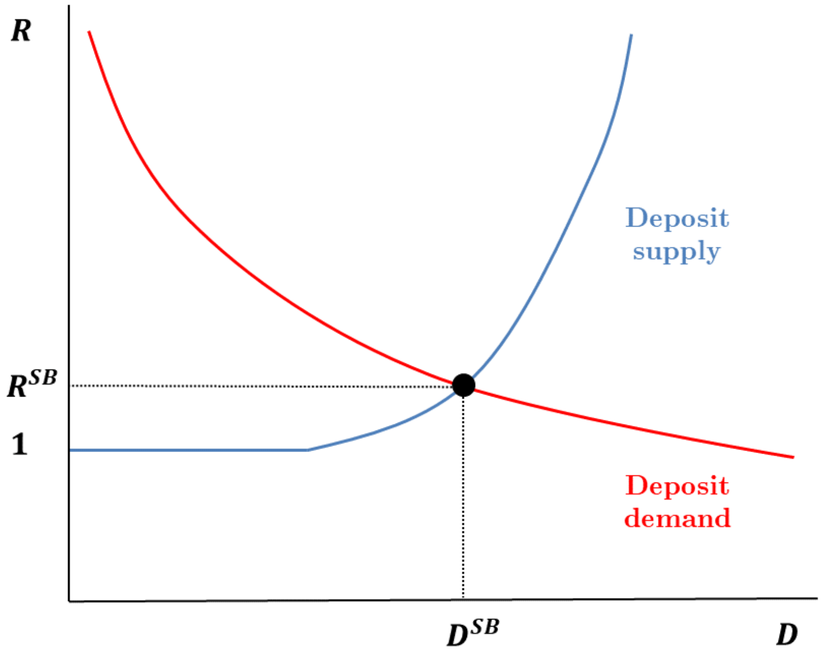
Figure 3.6 provides a graphical representation of the shadow banking strategy. The red line depicts banks' demand for deposits, which is inversely related to the asset price P_1. As deposits are used to purchase risky assets I_1, its downward slope reflects the positive relationship between I_1 and P_1 given by the asset supply schedule (3.3). The blue line depicts the supply of deposits by households. It is horizontal at the risk-free rate $R = 1$ when there is no early withdrawal or $\theta = 1$, but becomes upward sloping when a rise in P_1 reduces the liquidation value θ and drives households to require a higher interest rate in compensation as per (3.14). Under a shadow banking strategy, banks optimally invest up to the intersection of these two curves where the funding costs exceed the risk-free rate.

Traditional banking

Traditional banks choose $\left\{I_1^{TB}, I_2^{TB}, M_1^{TB}, M_2^{TB}, D^{SB}\right\}$ to maximize their expected profits

$$E\left[\Pi^{TB}\right] = (1-q)\left(\sigma_h I_1^{TB} + M_1^{SB}\right) + q(1-p)\left(\sigma_h I_2^{TB} + M_2^{TB}\right) - (1-qp)D^{TB}R^{TB}$$

Figure 3.6: **Shadow banking strategy**



Note: The deposit demand curve is attained by combining (3.3) and (3.17). Deposit supply stems from (3.14) and (C51).

subject to the budget constraints (3.17), (3.19) and the no-withdrawal constraint (3.21).[81]

Lemma 3.2 provides the solution to the traditional bank's problem.[82] It shows that, following bad news, traditional banks liquidate their risky asset holdings in the secondary market and re-allocate their portfolio toward cash to satisfy the no-withdrawal constraint.[83] Since the terms of trade between risky assets and cash depend on the secondary market price P_2, the no-withdrawal constraint also reduces banks' risky asset purchases I_1 in period 1 in line with P_2. Since the funding costs of traditional banks remain at the risk-free rate $\left(R^{TB} = 1\right)$, this creates an excess return that contributes to traditional bank profits (see Figure 3.7 for a graphical representation).

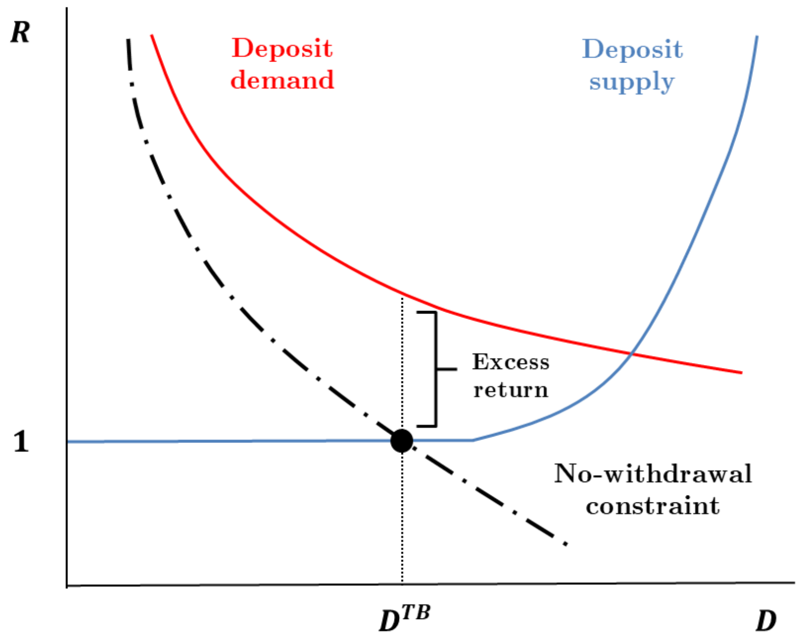
Lemma 3.2 *Under the restrictions $\mu \leq \bar{\mu}$, $\phi > \underline{\phi}$, the solution to the traditional bank's problem*

[81] Due to limited liability, traditional banks do not internalize the state with low payoff from assets.

[82] The indeterminacy of M_1^{TB} is due to two reasons. First, it does not contribute towards the no-withdrawal constraint, since each extra unit of cash corresponds to an extra unit of deposit. Second, the no-withdrawal constraint also prevents banks from converting cash to risky assets after bad news such that bank profits are not affected by M_1^{TB}.

[83] The complete liquidation of risky assets $\left(I_2^{TB} = 0\right)$ and the equivalence between first and second period asset prices $\left(P_1^{TB} = P_2\right)$ are due to the simplifying restrictions we have made in the interest of tractability. In Section 3.5, we show that the mechanism remains intact in a richer model which generates positive risky asset holdings after the sell-off and $P_1^{TB} > P_2$.

Figure 3.7: **Traditional banking strategy**



Note: The deposit demand curve is attained by combining (3.3) and (3.17). Deposit supply stems from (3.14) and (C51). The no-withdrawal constraint is given by (3.21), (3.24), and (3.25).

is

$$\begin{aligned} I_1^{TB} &= (A\alpha^\alpha)^{\frac{1}{1-\alpha}} P_2^{\frac{\alpha}{1-\alpha}} \\ I_2^{TB} &= 0 \qquad (3.24) \\ M_2^{TB} &= P_2 I_1^{TB} + M_1^{TB} \qquad (3.25) \\ E\left[\Pi^{TB}\right] &= (1-q)(\sigma_h - P_2) I_1^{TB} \end{aligned}$$

where $M_1^{TB} \geq 0$ *is indeterminate,* $\left(R^{TB}, \bar{V}\right)$ *are given by (3.16), and*

$$\frac{\partial E\left[\Pi^{TB}\right]}{\partial P_2} < 0 \; \forall \; P_2 > \alpha\sigma_h$$

Proof. Provided in Appendix C4. ■

3.3.2 Equilibrium

We solve for a rational expectations equilibrium such that all optimality conditions and constraints of banks, households, entrepreneurs and outside investors are satisfied, expectations are confirmed, and deposit and secondary markets clear such that

$$\gamma D^{SB} = d^{SB}$$
$$(1-\gamma) D^{TB} = d^{TB}$$
$$\phi = f\left(\gamma I_1^{SB} + (1-\gamma)\left(I_1^{TB} - I_2^{TB}\right)\right)$$

where $\gamma \in [0,1]$ is the share of banks that adopt a shadow banking strategy.

Furthermore, an equilibrium is characterized as 'interior' when traditional and shadow banks coexist such that γ falls within the range $0 < \gamma < 1$. In an interior equilibrium, banks are indifferent between shadow and traditional banking strategies such that (3.22) holds with equality

$$E\left[\Pi^{SB}\right] = E\left[\Pi^{TB}\right] - \tau \tag{3.26}$$

and may be interpreted as a free entry condition that determines γ in equilibrium.

We proceed as follows in our description of the equilibrium solution: Section 3.3.2.1 builds up on the intuition provided about bank strategies above by discussing their interaction with fire-sales. We focus on fire-sales due to their role in creating *strategic substitutabilities* in banks' decision to enter shadow banking, which is crucial for bringing about an interior equilibrium. Section 3.3.2.2 then provides the conditions under which an interior equilibrium arises and discusses the implications of a rise in the costs of commitment and a deepening of secondary markets.

3.3.2.1 Fire-sales and bank strategies

There is a two-way interaction between fire-sales and bank strategies. On the one hand, entry into shadow banking increases the excess supply of assets in the secondary market and exacerbates fire-sales.[84] On the other hand, fire-sales reduce the expected payoff from a shadow banking strategy relative to traditional banking and deter entry into the shadow banking sector.

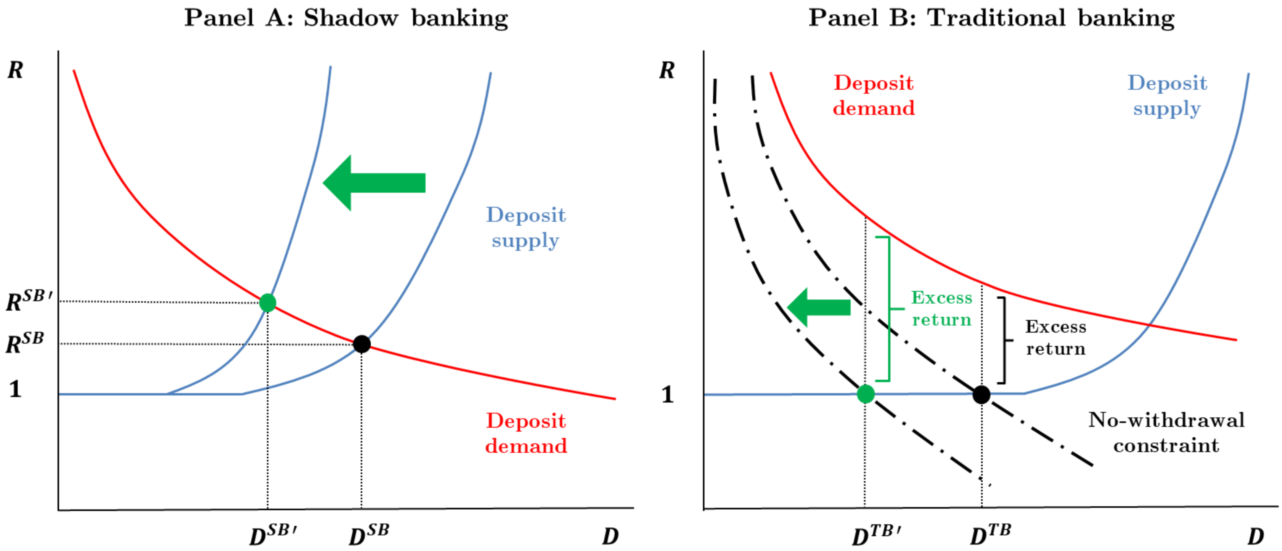
Figure 3.8 demonstrates the mechanism behind this. As shown in Panel A, a fall in the secondary market price P_2 reduces shadow banks' liquidation value during an early withdrawal. This leads to an upward pivot in the deposit supply curve, raising shadow banks' funding costs and reducing the expected payoff from shadow banking.

In contrast, a fall in P_2 leads to a rise in the expected payoff from traditional banking.[85] This is because it worsens the terms of trade between risky assets and cash after bad news, tightening the no-withdrawal constraint. As shown in Panel B, an inward shift in the constraint forces

[84] We use the phrases 'adopting a shadow banking strategy' and 'entry into shadow banking' interchangably.

[85] Strictly speaking, Lemma 3.2 indicates that a rise in profits occurs only in the region $P_2 > \alpha\sigma_h$. This reflects two conflicting effects on traditional bank profits: a rise in the return from risky asset purchases in period 1 versus a fall in the quantity purchased. For $P_2 > \alpha\sigma_h$, the former effect dominates and profits rise.

Figure 3.8: **Fall in the secondary market price**



Note: The deposit demand curve is attained by combining (3.3) and (3.17). Deposit supply stems from (3.14) and (C51). The no-withdrawal constraint is given by (3.21), (3.24), and (3.25).

traditional banks to reduce investment I_1 in risky assets in the first period. Since traditional banks' funding costs remain at the risk-free rate, this increases the excess return and expected payoff from traditional banking.

These interactions constitute a fire-sale externality since banks do not internalize the impact of their entry into shadow banking on the profitability of other banks. Moreover, since profitability affects entry incentives, fire-sale externalities create strategic substitutabilities in banks' decisions to adopt a shadow banking strategy. As the shadow banking sector grows, the fire-sale discount on risky assets gets larger (i.e. ϕ falls), reducing the payoff from shadow banking relative to traditional banking until we reach an equilibrium sector size where banks are indifferent between the two strategies.

3.3.2.2 Interior equilibrium

Proposition 3.2 provides the conditions for an interior equilibrium. It shows that there will be an interior equilibrium when commitment costs fall within the range $\tau \in (\underline{\tau}, \bar{\tau})$.

Proposition 3.2 *There is an interior equilibrium under the parameter restrictions* $\mu \leq \bar{\mu}$,

$\phi \in \left(\underline{\phi}, \bar{\phi}\right)$, $\alpha < 0.5$, $p < 1-\alpha$ and $\tau \in (\underline{\tau}, \bar{\tau})$ where

$$\underline{\tau} \equiv \begin{cases} \kappa\left[\frac{\sigma_h - 1}{q}\left(\frac{1-(1-q)\sigma_h}{q}\right)^{\frac{\alpha}{1-\alpha}} - \frac{\mu}{1+\mu}\sigma_h\left(1 - \frac{\mu}{1+\mu}(1-q)\sigma_h\right)^{\frac{\alpha}{1-\alpha}}\right] & \text{for } \mu < \frac{(p-q)(\sigma_h-1)}{qp(1-q)\sigma_h-(p-q)(\sigma_h-1)} \\ \begin{array}{l}\kappa\left(\sigma_h - \bar{\phi}\left[(1-p)\sigma_h + p\sigma_l\right]\right)\left(\bar{\phi}\left[(1-p)\sigma_h + p\sigma_l\right]\right)^{\frac{\alpha}{1-\alpha}} \\ -\kappa\frac{\mu\sigma_h}{1+\mu}\left((1-q)\frac{\sigma_h}{1+\mu} + q\bar{\phi}\left[(1-p)\sigma_h + p\sigma_l\right]\right)^{\frac{\alpha}{1-\alpha}}\end{array} & \text{otherwise} \end{cases}$$

$$\bar{\tau} \equiv \begin{cases} \kappa\left(\sigma_h\right)^{\frac{1}{1-\alpha}}\left[(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} - \frac{\mu}{1+\mu}\left(\frac{1-q}{1+\mu} + q\alpha\right)^{\frac{\alpha}{1-\alpha}}\right] & \text{for } \sigma_h \geq \frac{1}{1-qp(1-\alpha)} \\ \kappa\left[\frac{\sigma_h - 1}{qp}\left(\frac{1-(1-qp)\sigma_h}{pq}\right)^{\frac{\alpha}{1-\alpha}} - \frac{\mu}{1+\mu}\sigma_h\left((1-q)\frac{\sigma_h}{1+\mu} + \frac{1-(1-qp)\sigma_h}{p}\right)^{\frac{\alpha}{1-\alpha}}\right] & \text{otherwise} \end{cases}$$

such that $\kappa \equiv (1-q)\left(A\alpha^{\alpha}\right)^{\frac{1}{1-\alpha}}$, $\bar{\tau} > \underline{\tau} > 0$.

Proof. Provided in Appendix C6 ■

Finally, we briefly discuss the properties of this equilibrium. In an interior equilibrium, the free entry condition (3.26) pins down the size of the shadow banking sector γ and fire-sale discount ϕ through the interactions described in Section 3.3.2.1. Figure 3.9 illuminates the mechanism behind this by demonstrating the comparative statics of a rise in the commitment cost and a deepening of secondary markets under a numerical example.

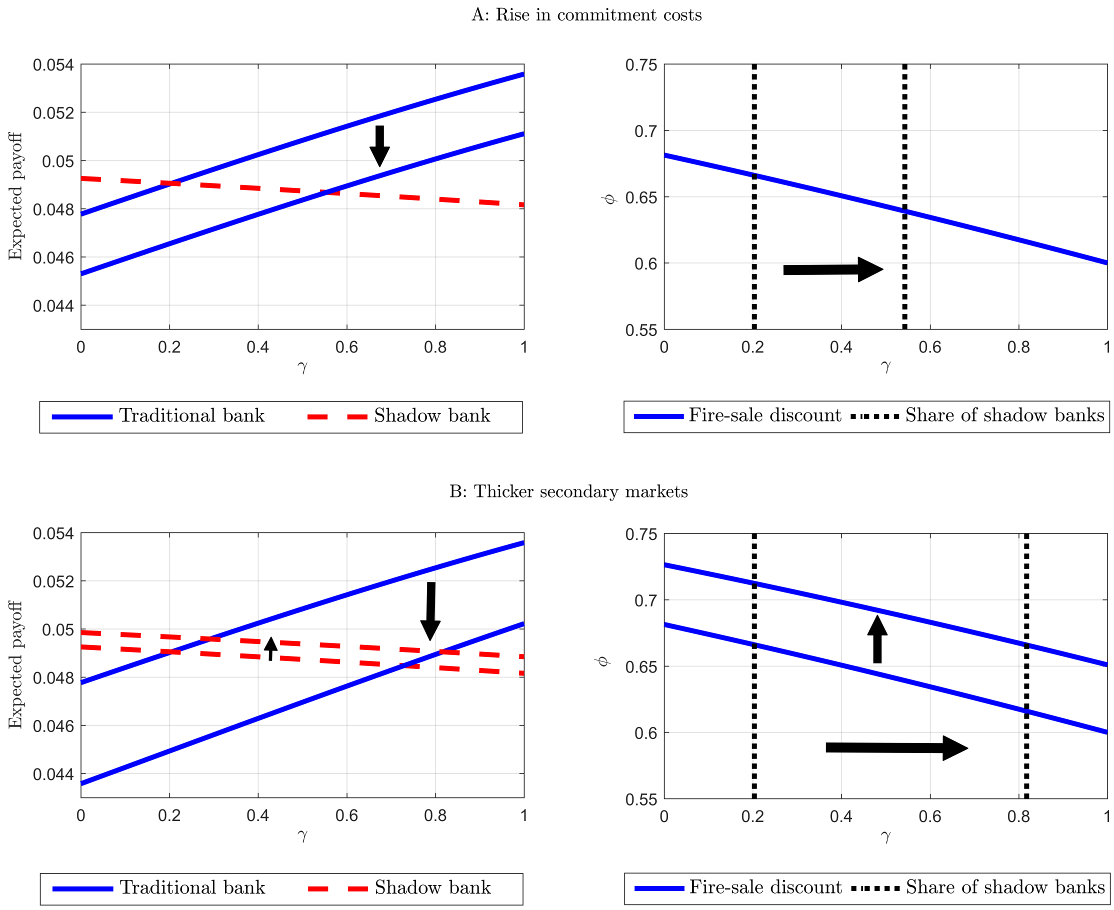
Observe that the expected payoff schedule for traditional (shadow) banking is upward (downward) sloping in γ in line with the intuition from Section 3.3.2.1. The equilibrium share of shadow banks is at the point where these two schedules intersect as per (3.26). We also plot this as a vertical bar along with the fire-sale schedule in order to deduce the equilibrium fire-sale discount.

Panel A shows that a rise in the cost of commitment τ causes a downward shift in the expected payoff schedule for traditional banking. At a given sector size γ, this makes shadow banking relatively profitable and leads to further entry into the sector. This in turn increases the excess supply of assets in the secondary market following bad news and exacerbates the fire-sale. Therefore, in equilibrium, a rise in the commitment cost increases the size of the shadow banking sector and the vulnerability of the economy to fire-sales.

Panel B shows the effects of an increase in the liquidity available in the secondary market due to a rise in the endowment of outside investors. At any given sector size γ, this reduces the fire-sale discount (i.e. a rise in ϕ) bringing about an upward shift in the fire-sale schedule. Consequently, the expected payoff schedule for traditional (shadow) banking shifts down (up) in line with Section 3.3.2.1, and there is entry into shadow banking until the new schedules intersect at a larger sector size.

It is important to note that the fire-sale discount returns to its initial value at the new equilibrium. As such, a thicker secondary market for assets increases the size of the shadow banking

Figure 3.9: **Numerical example**



Note: The numerical example correponds to the calibration $A = 1$, $\alpha = 1/3, q = p = \sigma_l = 1/2$, $\mu = \bar{\mu}$, and $\tau = 0.0825$. We parameterize the fire-sale function $f(.)$ according to Appendix C8 and calibrate $\widetilde{E}$ to get $\underline{\phi}$ as a lower bound and set $\kappa = 10\left((1-p)\,\sigma_h + p\sigma_l\right)^{-1}$. Panel A and B respectively display the effects of a small rise in τ and $\widetilde{E}$.

sector but does not alleviate the fire-sale. This finding stems from an essential property of the interior equilibrium: the fire-sale discount is implicitly determined by the free entry condition (3.26). Note that these results also apply to policy interventions aimed at alleviating fire-sales such as asset purchases and bailouts of financial institutions that would have contributed to the excess supply of assets. We analyze policy interventions in greater detail in Section 3.6.

3.4 A model with liquidity risk

In this section, we extend the simple model to a setting with liquidity risk and a richer asset space. We introduce liquidity risk by allowing for the possibility of bank-runs as in Diamond and Dybvig (1983). Bank-runs are distinct from early withdrawals in that they are driven by self-fulfilling expectations in the presence of a liquidity shortfall ($\theta < 1$) in bank balance sheets, rather than an optimal response to bad news.

We also expand the asset space to allow for three different asset types, liquid, safe, and risky, which we respectively denote with (λ, s, r). The risky asset is identical to the (non-cash) asset in the simple model. The safe and liquid assets both yield a unit payoff with certainty, but they differ in that the safe asset matures in period 3 while the liquid asset yields its payoff in period 2.

The richer asset space serves two purposes. First, it allows us to consider the conditions under which safe assets are endogenously liquid due to secondary markets. A priori, it is not clear whether safe assets would be subject to a fire-sale since purchases by traditional banks (which re-allocate their portfolios towards safe assets) may offset sales by shadow banks.

Second, a richer asset space permits us to consider the interplay between fire-sales, market discipline, and liquidity risk. On the one hand, without a fire-sale on safe assets, portfolio re-allocation required to satisfy the no-withdrawal constraint increases traditional banks' liquidation values ($\theta^{TB} = 1$) and reduces their vulnerability to bank-runs. On the other hand, when there is a fire-sale on safe assets, liquidity risk leads to a rise in bank funding costs and undermines market discipline such that traditional banks may also take on solvency risk.

In the interest of brevity, we only describe the aspects of the model that differ from Section 3.3.[86] The remainder of the section is organized as follows: Section 3.4.1 provides further details about bank-runs. Section 3.4.2 extends secondary markets and outside investors to a framework with multiple assets. Finally, Section 3.4.3 presents analytical results pertaining to the relationship between the fire-sale on safe assets, market discipline, and liquidity risk.

3.4.1 Bank-runs

When a bank has a liquidity shortfall $\theta < 1$, sequential service in withdrawals leads to the emergence of a bank-run equilibrium where households find it optimal to withdraw their deposits given that everyone else is withdrawing. We resolve this multiplicity with the use of sunspots. Specifically, we let ε_ξ be be a random variable drawn from a uniform distribution on the unit interval at the beginning of period 2 and $\bar{\xi} \in [0, 1]$ a constant threshold. When $\varepsilon_\xi \leq \bar{\xi}$, household sentiments coordinate on a bank-run. Provided that there is a liquidity shortfall, these sentiments become self-fulfilling and the bank suffers from a run. Since ε_ξ is uniformly distributed on a unit interval, the probability of a bank run is then simply given by

$$\xi = \begin{cases} 0 \text{ for } \theta = 1 \\ \bar{\xi} \text{ for } \theta < 1 \end{cases}$$

[86] See Appendix C9 for a complete specification of the model.

Only traditional banks are affected by bank-runs as there is no liquidity shortfall after good news and shadow banks face an early withdrawal after bad news.[87] Moreover, we assume that the sunspot realization is idiosyncratic to each bank such that ξ may be interpreted as the share of traditional banks liquidated in a bank-run.[88]

Finally, note that it is straightforward to introduce a more sophisticated specification for the bank-run probability. We adopt this simple framework as it permits us to isolate the role of banks' portfolio strategies and fire-sales in bringing about a liquidity shortfall in the first place. In Appendix C10, we approximate the global games solution of Goldstein and Pauzner (2005) by depicting $\bar{\xi}$ as a negative function of the liquidation value θ. The qualitative results from our mechanism remain the same.

3.4.2 Secondary market

We extend the secondary market described in Section 3.3.1.2 to a framework with multiple assets. Since the liquid asset yields its payoff in period 2, only safe and risky assets are traded in the secondary market. The excess supply of each asset is given by the expressions

$$\widetilde{I}(i) = \gamma I_1^{SB}(i) + (1-\gamma)\left(I_1^{TB}(i) - (1-\xi)\, I_2^{TB}(i)\right) \geq 0 \;\forall\; i \in \{s, r\}$$

and sold to a common set of risk neutral outside investors.

Outside investors allocate their endowment $\widetilde{E}$ between purchases of safe and risky assets and an outside project $\widetilde{K}$ that yields a safe payoff $g\left(\widetilde{K}\right)$ where $g'(.) > 0$, $g''(.) < 0$. Their first order conditions indicate that there is a common fire-sale discount $\phi \in [0,1]$ for the two assets, and it is defined implicitly by the expression

$$\phi = \frac{1}{g'\left(\widetilde{E} - \phi\left[(1-p)\,\sigma_h + p\sigma_l\right]\widetilde{I}(r) - \phi\widetilde{I}(s)\right)} \tag{3.27}$$

which suggests that a rise in the total excess supply of safe and risky assets leads to a decrease in ϕ. Crucially, however, an asset type is only subject to the fire-sale discount when it is in excess supply such that its marginal buyer is an outside investor. Therefore, secondary market

[87]To be precise, we assume that early withdrawals and bank-runs take place simultaneously so that there is no need to consider the bank run equilibrium for shadow banks.

[88]This assumption streamlines the exposition by preventing the outcome in the second period from diverging between bank-run and no bank-run. With an aggregate sunspot, uncertainty would immediately be resolved in the bank-run equilibrium. The non-bank run equilibrium would only differ from the equilibrium considered here in that the excess supply of assets in the secondary market would be somewhat lower.

prices depend on an asset's own excess supply such that

$$P_2(s) = \begin{cases} 1 & \text{for } \widetilde{I}(s) = 0 \\ \phi & \text{otherwise} \end{cases}$$

$$P_2(r) = \begin{cases} (1-p)\sigma_h + p\sigma_l & \text{for } \widetilde{I}(i) \leq 0 \\ \phi[(1-p)\sigma_h + p\sigma_l] & \text{otherwise} \end{cases}$$

3.4.3 Analytical results

Proposition 3.3 shows that the fire-sale on safe assets plays a definitive role in shaping the relationship between market discipline and liquidity risk.

Proposition 3.3 *In equilibrium, traditional banks commit to a minimum recovery rate*

$$\bar{V} = 1 - \frac{q}{p} \frac{\xi\left(1-\theta^{TB}\right)}{1 - q\left(1 - \xi\left(1-\theta^{TB}\right)\right)} \tag{3.28}$$

and the interest rate on their deposits is given by

$$R^{TB} = 1 + \frac{q}{1-q}\xi\left(1-\theta^{TB}\right) \tag{3.29}$$

With $P_2(s) = 1$, traditional banks have a liquidation value $\theta^{TB} = 1$. This leads to $\xi = 0$ and (3.28), (3.29) yield

$$\bar{V} = R^{TB} = 1$$

With sufficiently low $P_2(s) < 1$, traditional banks have a liquidity shortfall $\theta^{TB} < 1$ which leads to $\xi > 0$. (3.28), (3.29) then yield

$$\bar{V} < 1 < R^{TB}$$

Proof. Provided in Appendix C7 ■

When there is no fire-sale on safe assets such that $P_2(s) = 1$, satisfying the no-withdrawal constraint

$$\sigma_l I_2^{TB}(r) + I_2^{TB}(s) \geq \bar{V} R^{TB} D^{TB} \tag{3.30}$$

leads to a liquidation value

$$\theta^{TB} = \min\left[1, \frac{P_2(r) I_2^{TB}(r) + P_2(s) I_2^{TB}(s)}{D^{TB}}\right] = 1 \tag{3.31}$$

since risky assets never trade at a price below their payoff under weak fundamentals ($P_2(r) \geq \sigma_l$). This in turn eliminates the possibility of bank-runs such that $\xi = 0$. Without liquidity risk, market discipline through the threat of an early withdrawal ensures that there is also no solvency risk ($\bar{V} = 1$) and traditional banks collect deposits at the risk-free rate as in Section 3.3.1.3.

With a sufficiently large fire-sale on safe assets, on the other hand, traditional banks have a liquidity shortfall ($\theta^{TB} < 1$) despite satisfying the no-withdrawal constraint. This leaves traditional banks vulnerable to bank-runs and creates a vicious cycle between rising deposit interest rates and solvency risk. Households anticipate that they may not be repaid fully in the event of a bank run and demand higher deposit rates as compensation. The rise in deposit rates in turn weakens the threat of early withdrawal as households stand to lose more in terms of interest foregone. This leads to a decline in the minimum recovery rate $\bar{V}$, allowing traditional banks to take on solvency risk. Finally, the increase in solvency risk brings about a further rise in deposit rates and completes the vicious cycle.

In the next section, we present numerical results that highlight the role of a large shadow banking sector in causing a fire-sale on safe assets, which leaves traditional banks illiquid and undermines market discipline through the mechanism described above.

3.5 Numerical results

This section provides numerical results from the model with liquidity risk. It proceeds as follows: Section 3.5.1 describes the calibration which targets the United States over the recent financial crisis. Section 3.5.2 presents and discusses the results from a numerical simulation.

3.5.1 Calibration

Table 3.2 reports the calibrated parameters. The payoff of risky asset under weak fundamentals is set to $\sigma_l = 0.21$ in line with the average recovery rate from junior debt and σ_h is backed out using the normalization of expected payoffs given in (3.1).[89] The probabilities (q, p) are calibrated to the frequency of recessions and deep recessions (conditional on a recession) in the United States.[90]

Regarding the entrepreneurs, we calibrate the output elasticity of capital to the standard Cobb-Douglas value $\alpha = 1/3$ and normalize productivity to $A = 1$. For the mark-up, we adopt a parameterization that approximates monopolistic competition

$$\mu = \upsilon \frac{1 - \alpha}{\alpha}$$

[89] Security classes that have less seniority than 70% of total liabilities are defined as junior.

[90] We use business cycle data from the NBER which covers the period December 1854-June 2017 at a monthly frequency. We label as 'deep recessions' the contractionary episodes of 1873-79 (the Long Depression), 1929-33 (the Great Depression), and 2008-09 (the Great Recession).

Table 3.2: **Calibration**

,

Parameter	Value	Description	Source
σ_l	0.21	Low payoff from risky assets	Moody's Investors Service (2007)
q	0.41	Prob. of bad news	NBER
p	0.22	Prob. of weak fundamentals	NBER
α	0.33	Cobb-Douglas parameter	-
A	1.00	Productivity	-
μ	0.18	Mark-up	World Bank
$\widetilde{E}$	2.80	Outside investor endowment	Moody's, Federal Reserve Board
z	10.0	Outside investment parameter	Moody's, Federal Reserve Board
$\bar{\xi}$	0.15	Bank-run probability	Federal Reserve Bank of St. Louis
τ	0.11	Commitment cost	Financial Stability Board (2017)

where $\upsilon \in [0,1]$ represents banks' market share. We then calibrate the mark-up to a value consistent with the 5-bank asset concentration in the United States over 2007-2010.

In the secondary market, we parameterize the payoff function for outside investments to $g\left(\widetilde{K}\right) = z^{-1}\ln\left(\widetilde{K}\right)$ where z is a constant such that (3.27) becomes

$$\phi = \frac{z\widetilde{E}}{1 + z\left[\left(\left(1-p\right)\sigma_h + p\sigma_l\right)\widetilde{I}\left(r\right) + \widetilde{I}\left(s\right)\right]}$$

Our calibration strategy for $\left(\widetilde{E}, z\right)$ targets the rise in the yield spreads between AAA-rated seasoned corporate bonds and the effective Federal Funds Rate during the financial crisis. Specifically, we back out a percentage decline in bond prices $\Delta\hat{P}/\hat{P}$ from the difference between the peak spread in November 2008 and the average spread for the pre-crisis period between January 2016 and June 2017. We then calibrate $\left(\widetilde{E}, z\right)$ to generate an equilibrium fire-sale discount that matches the decline in the price of safe assets under bad news to $\Delta\hat{P}/\hat{P}$.[91]

Similarly, the calibration for the bank-run probability $\bar{\xi}$ targets TED spreads at the peak of the financial crisis. To do this, we construct a hypothetical interest rate for traditional bank deposits collected after bad news but before the realization of bank-runs. With risk neutral households, this is given by

$$R_2^{TB} = \frac{1}{1-\bar{\xi}}\frac{1-\bar{\xi}\theta^{TB}}{1-p\left(1-\bar{V}\right)} \tag{3.32}$$

[91] Since $P_1\left(s\right)$ is bank-specific and best interpreted as origination costs in the absence of first period asset trade, our measure for the decline in safe asset prices refers to $P_2\left(s\right)$ relative to their expected payoff 1.

and we calibrate $\bar{\xi}$ to match R_2^{TB} to the average TED spread in the last quarter of 2008.[92]

Finally, the calibration for the commitment cost τ targets the pre-crisis ratio of shadow bank assets to total bank assets, which is approximately 63%. Accordingly, we set $\tau = 0.11$ which amounts to just below 7% of the value of a traditional bank's assets in period 1.

3.5.2 Results

Figure 3.10 plots the numerical solution across a range $\gamma \in [0, 1]$ of shadow banking sector sizes. The equilibrium sector size is denoted by the vertical bar labelled γ^* where traditional and shadow banking strategies yield the same expected payoff. The first panel of Figure 3.10 indicates that there is a unique and globally stable interior equilibrium where shadow and traditional banks coexist. This reflects the strategic substitutabilities between banking strategies described in Section 3.3.2.

Entry into shadow banking exacerbates the fire-sale after bad news (see Panel 2). This increases the interest rates on shadow bank deposits (see Panel 4) and reduces the expected payoff from shadow banking. In contrast, traditional bank profits rise as the fire-sale increases the excess return from the binding no-withdrawal constraint. Therefore, the shadow banking sector grows until banks are indifferent between the two strategies at the equilibrium sector size γ^*.

The bar labelled $\bar{\gamma}$ denotes the threshold sector size above which safe assets suffer from a fire-sale. Under our calibration, high commitment costs lead to $\gamma^* > \bar{\gamma}$ such that there is a fire-sale on safe assets in equilibrium. Panel 3 shows that this reduces the liquidation value of traditional banks, leaving them vulnerable to bank-runs. As explained in Section 3.4.3, liquidity risk increases deposit rates R^{TB} (see Panel 4) and undermines market discipline on traditional banks. Accordingly, Panel 5 shows that the minimum recovery rate $\bar{V}$ declines below full repayment for $\gamma > \bar{\gamma}$ such that traditional banks have insolvency risk in equilibrium.

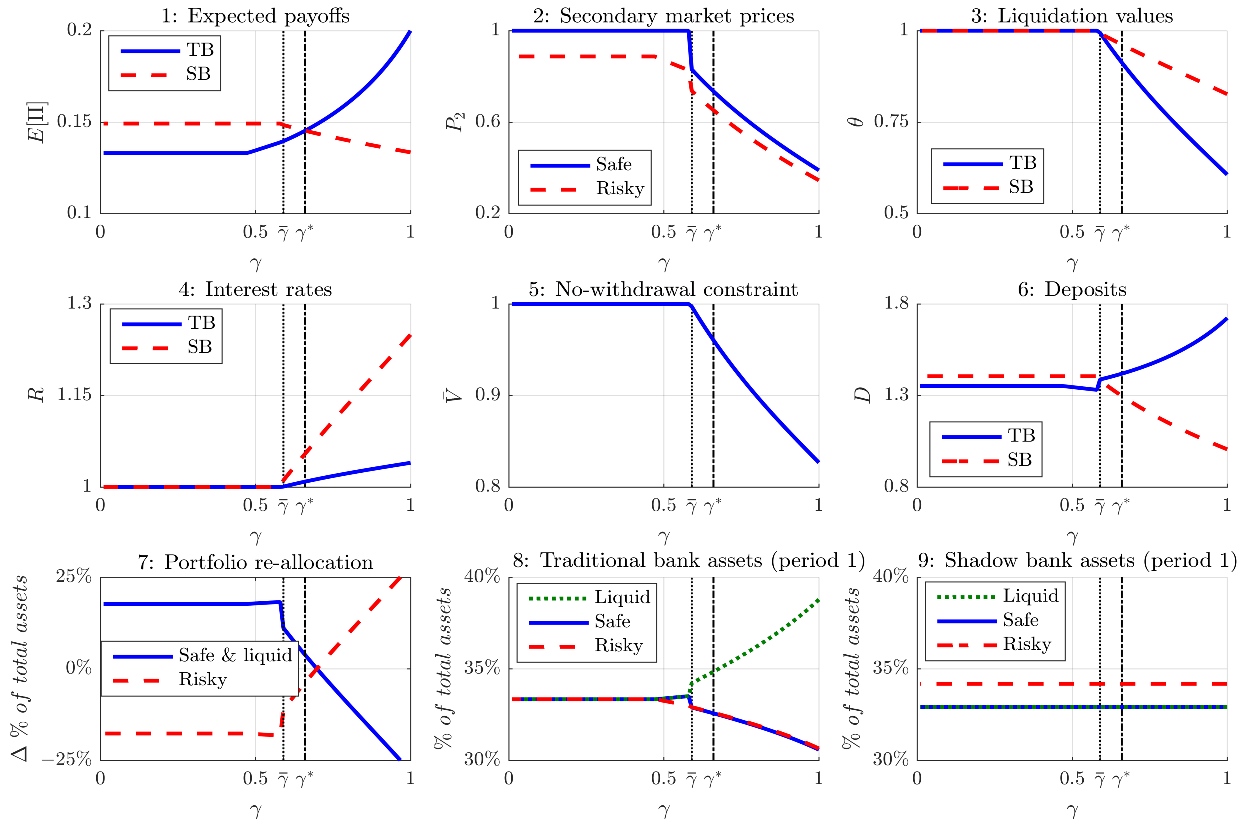
Note that, with lower commitment costs, the expected payoff from traditional banking is higher at all sector sizes such that we may have a smaller shadow banking sector $\gamma^* \leq \bar{\gamma}$ in equilibrium and no fire-sale on safe assets.[93] Panel 3 shows that traditional banks would then have no liquidity shortfall and hence no vulnerability to bank-runs. Without liquidity risk, traditional banks borrow at the risk-free rate and the no-withdrawal constraint imposes market discipline with $\bar{V} = 1$ such that traditional banks have no solvency risk either.

Panel 6 shows the evolution of shadow and traditional bank deposits across γ. As γ rises and the fire-sale deepens, shadow banks respond to the rise in their funding costs by reducing

[92] See Figure 3.4 for the evolution of TED spreads. We use TED spreads instead of spreads on deposit rates to exclude the effects of deposit insurance guarantees. Note also that (3.32) is conditional on a liquidity shortfall, which is the case in equilibrium.

[93] See Figure 3.9 for the comparative statics of the commitment cost and fire-sale discount in the simple model.

Figure 3.10: **Numerical results**



Note: Expected payoffs are inclusive of the commitment cost τ. Total assets in period 1 and 2 are respectively defined as $\bar{I}_1 \equiv \sum_{i\in\{\lambda,s,r\}} I_1(i)$ and $\bar{I}_2 \equiv \sum_{i\in\{s,r\}} P_2(i)\, I_2(i)$. Panel 7 plots $\left(\bar{I}_2^{TB}\right)^{-1} P_2(s)\, I_2^{TB}(s) - \left(\bar{I}_1^{TB}\right)^{-1} \sum_{i\in\{\lambda,s\}} I_1^{TB}(i)$ for safe and liquid assets, and $\left(\bar{I}_2^{TB}\right)^{-1} P_2(r)\, I_2^{TB}(r) - \left(\bar{I}_1^{TB}\right)^{-1} I_1^{TB}(r)$ for risky assets. Panel 8 plots $I_1^{TB}(i)/\bar{I}_1$ respectively for $i = \{\lambda, s, r\}$ and Panel 9 does the same for shadow banks.

their deposits. Traditional banks, on the other hand, expand their balance sheets due to the relaxation of the no-withdrawal constraint. This is consistent with observations in Fact 1 of Section 2.2 which indicate that traditional bank balance sheets expanded at a faster rate in 2002-07 when the shadow banking sector was growing rapidly than in 2008-15 when the shadow banking sector was stagnant.[94]

Panel 7 demonstrates portfolio re-allocation by traditional banks after bad news. When the shadow banking sector is small and market discipline intact, traditional banks re-allocate up to 18% of their portfolio from risky assets to safe and liquid assets in order to prevent an early withdrawal. As the shadow banking sector grows and market discipline is undermined, traditional banks reduce the extent of their re-allocation away from risky assets and eventually the direction is reversed. In equilibrium, there is a re-allocation of approximately 4% away from

[94] Note that, although Panel 6 indicates that, in equilibrium, an individual traditional bank has a larger balance sheet than an individual shadow bank, this does not contradict Figure 3.1, which is at the aggregate level. In fact, we calibrate τ to target the ratio of shadow bank assets to traditional bank assets in 2007.

risky assets. Compared to the observations in Fact 4 of Section 3.2, this predicts the direction of portfolio re-allocation correctly, but falls somewhat short of the observed amount of 6% to 8% (see Figure 3.4). It is possible that this discrepancy reflects the impact of macroprudential regulation and additional liquidity provided by the Federal Reserve.

Finally, Panels 8 and 9 display the investment strategies of traditional and shadow banks in period 1. When there is no fire-sale on safe assets ($\gamma < \bar{\gamma}$), market discipline imposed by the no-withdrawal constraint forces traditional banks to behave as if they internalize asset payoffs under weak fundamentals. Therefore, they devote an equal share of their investment to each asset type. In response to a fire-sale on safe assets, traditional banks increase their holdings of liquid assets to prop up their liquidation value. While this increases the funds available to them after bad news, their reaction is not strong enough to keep them solvent under weak fundamentals because of the decline in the minimum recovery rate $\bar{V}$ (as shown in Panel 5).

Shadow banks, on the other hand, skew their investment towards risky assets since they only internalize the states after good news due to limited liability. Without the ability to commit, they also take their borrowing costs R^{SB} as given and hence do not change their asset composition in response to fire-sales.

3.6 Policy analysis

This section considers policy interventions aimed at fostering financial stability. The numerical results in Section 3.5.2 demonstrate that, with sufficiently high commitment costs, the shadow banking sector expands to a size that is systemically risky in the sense that its collapse gives rise to both liquidity and solvency risk for traditional banks.

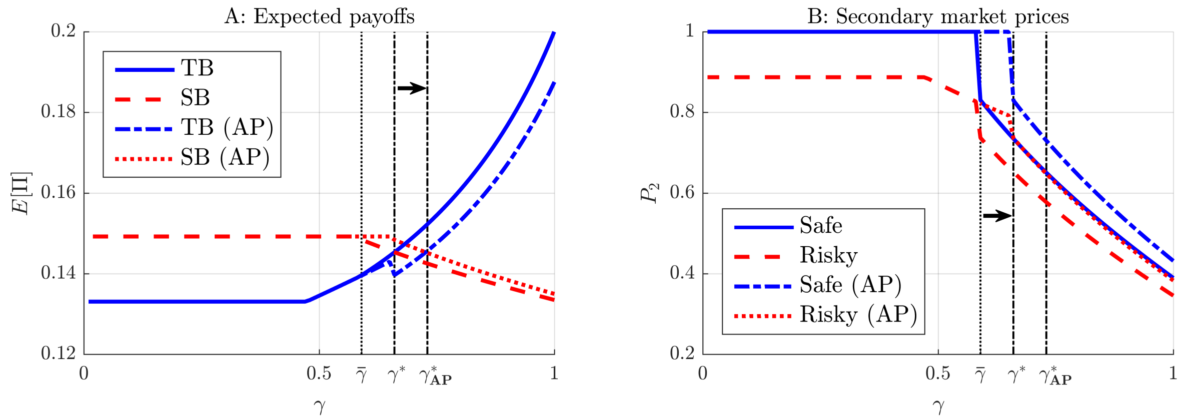
We consider four distinct interventions that aim to offset this. Section 3.6.1 considers asset purchases in the secondary market. Section 3.6.2 proposes a tax on shadow bank profits. Section 3.6.3 considers liquidity provision to traditional banks. Finally, Section 3.6.4 evaluates proposals to ringfence the traditional banking sector with a combination of deposit insurance guarantees and macroprudential regulation.

3.6.1 Asset purchases

The government can lean against fire-sales by purchasing assets in the secondary market. For the sake of simplicity, we only consider safe asset purchases such that their excess supply becomes

$$\widetilde{I}\left(s\right) = \gamma I_1^{SB}\left(s\right) + \left(1-\gamma\right)\left(I_1^{TB}\left(s\right) - \left(1-\xi\right) I_2^{TB}\left(s\right)\right) - I^{AP}$$

Figure 3.11: **Asset purchases**



Note: Expected payoffs are inclusive of the commitment cost τ. AP refers to the outcome under asset purchases.

where $I^{AP} > 0$ refers to asset purchases by the government.[95] Figure 3.11 shows the outcome of an intervention that aims to eliminate the fire-sale on safe assets by setting I^{AP} to absorb their excess supply at the equilibrium sector size γ^*.

The outcome of this intervention is identical to the deepening of secondary markets considered in Section 3.3.2.2. It reduces the fire-sale discount (i.e. increases ϕ) at any given sector size and is successful in eliminating the fire-sale on safe assets at the initial equilibrium γ^*. In other words, the intervention shifts the fire-sale threshold $\bar{\gamma}$ to γ^* (see Panel B). However, by alleviating the fire-sale, the intervention also reduces the funding costs of shadow banks and increases the expected payoff from shadow banking relative to traditional banking (see Panel A). This leads to entry into the shadow banking sector until we reach the new equilibrium sector size $\gamma^*_{AP} > \gamma^*$.

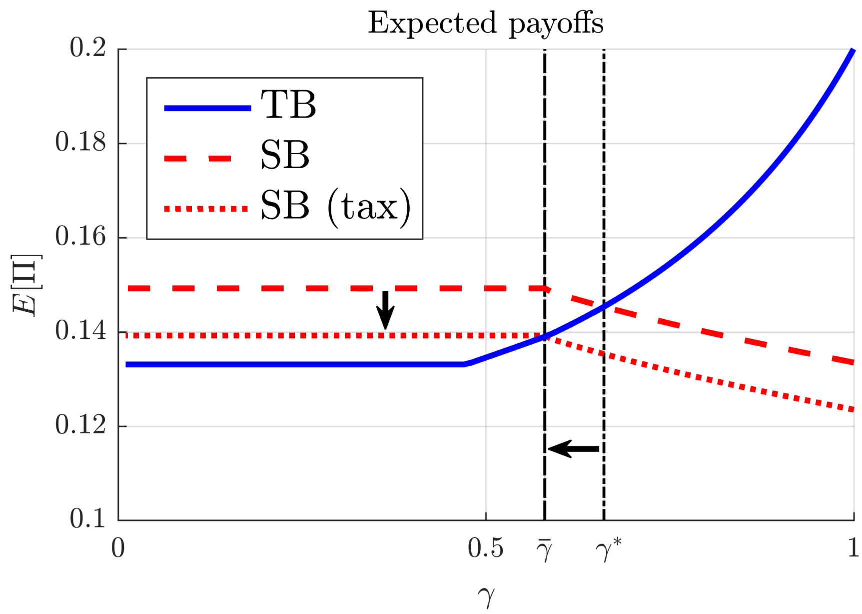
It is not a coincidence that in the new equilibrium γ^*_{AP}, the fire-sale discount (and all variables other than the sector size) takes the same value as in the initial equilibrium γ^*. This is because, as explained in Section 3.3.2.1, the fire-sale discount is implicitly determined by the free entry condition (3.26).

Our analysis then leads to a crucial insight: policy interventions in secondary markets are ineffective when there is free entry into shadow banking. This insight also applies in the opposite direction. For example, a tax on secondary market transactions deepens the fire-sale and reduces shadow bank profits at a given sector size, but both of these effects are reversed by exit from the shadow banking sector.

Finally, is important to stress that entry into shadow banking is driven by the ex-ante anticipation of asset purchases. In period 2, the size of the shadow banking sector is already

[95] Allowing for risky asset purchases leads to no discernible changes in the results.

Figure 3.12: **Tax on shadow bank profits**



Note: Expected payoffs are inclusive of the commitment cost τ and tax T.

determined and a surprise asset purchase scheme would be successful in alleviating the fire-sale on safe assets and reducing both liquidity and solvency risk. This creates the potential for time inconsistency issues as policymakers would naturally find it tempting to intervene once the fire-sale is underway.

3.6.2 Tax on shadow bank profits

The second policy intervention we consider is the taxation of shadow bank profits with the purpose of deterring entry into the shadow banking sector. This can be considered as a Pigouvian tax since entry into shadow banking imposes a negative externality on the remainder of the financial sector through its contribution to fire-sales.

We consider a constant tax T such that the free entry condition becomes

$$E\left[\Pi^{SB}\right] - T = E\left[\Pi^{TB}\right] - \tau$$

Figure 3.12 shows the outcome under a tax level $0 < T < \tau$ that reduces the equilibrium sector size of shadow banks to the fire-sale threshold $\bar{\gamma}$. The tax effectively shifts down the expected payoff schedule from shadow banking and brings about an equilibrium without a fire-sale on safe assets. Since this is a tax on profits, the equilibrium allocation at any given sector size γ is identical to the numerical results in Figure 3.10. Therefore, reducing the equilibrium sector size to $\bar{\gamma}$ eliminates liquidity risk in the traditional banking sector, which strengthens market discipline such that there is also no solvency risk.

Note that the effect of the tax is equivalent to a decrease in the commitment cost τ. Therefore, when taxing shadow bank profits is not feasible, the same outcome can also be achieved with a transfer to traditional banks.[96] Alternatively, shadow bank profits may be reduced indirectly with a tax on their liabilities D^{SB}. In a less stylized set up, this would correspond to a tax on the funding instruments most widely used by shadow banks such as ABCPs.

3.6.3 Liquidity provision

Recall from Fact 4 in Section 3.2 that traditional bank liabilities sharply increased in the second half of 2008. This increase was driven by Federal Reserve funding and a rise in deposits protected by deposit insurance guarantees. In this section, we show that liquidity support of this form may effectively ringfence the traditional banking sector from the financial instability caused by shadow banks.

We consider a liquidity provision scheme that permits traditional banks to borrow L from the central bank at the risk-free rate in the second period. The liquidation value of traditional banks can then be written as

$$\theta^{TB} = \min\left[1, \frac{P_2(r) I_1^{TB}(r) + P_2(s) I_1^{TB}(s) + I_1(\lambda) + L}{D^{TB}}\right]$$

and two conditions must be satisfied for liquidity provision to eliminate both solvency and liquidity risk. First, the amount of liquidity provided should be sufficient to offset liquidity shortfalls such that

$$L \geq \underline{L} \equiv D^{TB} - P_2(r) I_1^{TB}(r) - P_2(s) I_1^{TB}(s) - I_1(\lambda)$$

This eliminates the possibility of bank-runs ($\xi = 0$) and liquidity risk.

Second, liquidity borrowed from the central bank should be fully collateralized.[97] This rules out default on the central bank and makes depositors effectively junior to the central bank. The no-withdrawal constraint then becomes

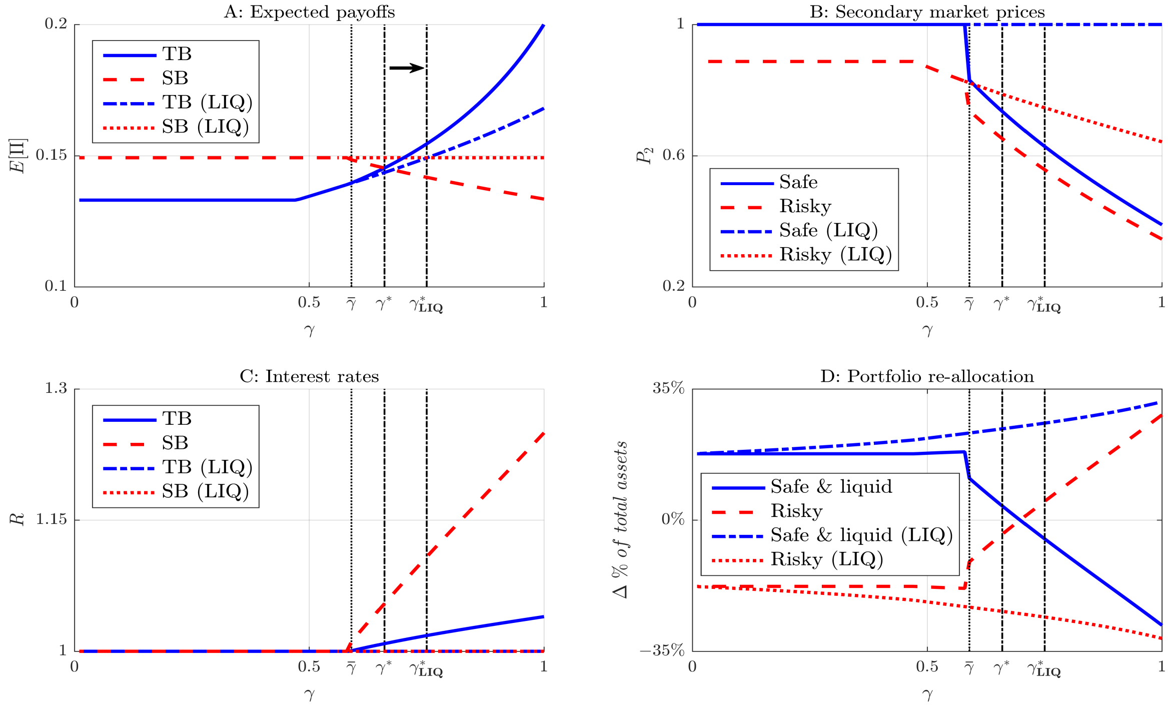
$$\sigma_l I_2^{TB}(r) + I_2^{TB}(s) \geq L + \bar{V} D^{TB} R^{TB}$$

such that market discipline is not adversely affected by liquidity provision. The results from Proposition 3.3 then follow through: in the absence of liquidity risk, the threat of an early

[96] Quantitatively, our model indicates that the necessary transfer amounts to 9% of commitment costs, which is equivalent to approximately 0.6% of the value of a traditional bank's portfolio in period 1.

[97] This places an upper bound on the amount of liquidity provided $L \leq \bar{L} = \sigma_l I_2^{TB}(r) + I_2^{TB}(s)$ where $\bar{L} \geq \underline{L}$.

Figure 3.13: **Liquidity Provision**



Note: Expected payoffs are inclusive of the commitment cost τ.

withdrawal eliminates solvency risk ($\bar{V} = 1$) and interest rates on traditional bank deposits fall to the risk-free rate ($R^{TB} = 1$).

Figure 3.13 shows the equilibrium outcome under liquidity provision in comparison to the baseline results in Section 3.5.2. Panel B shows that, in addition to ringfencing the traditional banking sector, liquidity provision eliminates the fire-sale on safe assets. This is because, with $P_2(s) < 1$, traditional banks take advantage of an arbitrage opportunity by borrowing from the central bank at the risk-free rate and purchasing safe assets at a discount.[98] They continue to do this until there is no excess supply of safe assets such that $P_2(s) = 1$. Panel D shows that these purchases also increase the extent of portfolio reallocation towards safe and liquid assets.

By alleviating the fire-sale, liquidity provision indirectly reduces the funding costs of shadow banks. This in turn increases the expected payoff from shadow banking, leading to further entry into the sector (see Panel A). Therefore, liquidity provision leads to a rise in the equilibrium size of the shadow banking sector, but this has no adverse effects on traditional banks.

[98]This arbitrage opportunity is not present in the baseline model. Without liquidity provision or deposit insurance guarantees, traditional banks may only collect funds in the second period by offering interest rates $R_2^{TB} > 1$ given by (3.32). They find it prohibitively costly to do so.

3.6.4 Deposit insurance and macroprudential regulation

The liquidity provision scheme described in the previous section is equivalent to the combination of deposit insurance with regulation that imposes a minimum recovery rate $\bar{V} = 1$. The two policy interventions only differ with respect to the role of market discipline. As explained above, liquidity provision is designed to take advantage of market discipline while securing traditional banks from liquidity risk. In contrast, eliminating liquidity risk through deposit insurance also removes the no-withdrawal constraint on traditional banks since depositors never stand to take a loss. Market discipline then has to be replaced with macroprudential regulation.

3.7 Conclusion

We have presented a model of the financial sector in which shadow banking emerges endogenously as an alternative banking strategy. A key aspect of the model is that depositors re-optimize in response to revisions in expectations about asset returns. To prevent early withdrawals by their depositors, traditional banks optimally commit to a safe portfolio strategy, while shadow banks combine high risk-taking with the prospect of an early liquidation after bad news.

Two important insights emerge as a consequence. First, costly commitment and fire-sale externalities bring about an equilibrium where ex-ante identical banks optimally cluster into shadow and traditional banking strategies. The size of the shadow banking sector increases in the cost of commitment and the availability of liquidity in the secondary market. Second, when the shadow banking sector is large, the liquidation of shadow banks leave traditional banks susceptible to liquidity risk. This increases deposit rates offered by traditional banks and weakens market discipline on them, engendering greater risk-taking and a rise in solvency risk.

The model naturally provides novel insights for policy design. Policy interventions have significantly different implications when the adjustment on the size of the shadow banking sector is taken into account. Secondary market interventions, such as asset purchases by the government, are effective in alleviating the fire-sale ex-post, but their ex-ante expectation fuels further growth of the shadow banking sector in a manner that exactly offsets these gains. This leads to time inconsistency issues for policymakers which may find it tempting to intervene once the fire-sale is underway.

The findings regarding the destabilizing consequences of a large shadow banking sector lend support to taxation of shadow bank profits (or equivalently a transfer to traditional banks) with the purpose of reducing the size of the shadow banking sector to a level compatible with financial stability. Alternatively, collateralized liquidity support or a combination of deposit insurance guarantees and macroprudential regulation are effective in ringfencing the traditional banking sector but bring about an expansion in shadow banking.

Finally, it is worth noting that the mechanism considered in this chapter is more general than its application to shadow banking and the 2007-09 financial crisis. In economies without credible deposit insurance guarantees and strict enforcement of banking regulation, financial intermediation strategies that combine high risk-taking with an unstable funding structure may exist within the commercial banking sector. One clear example of this is the United States during the National Banking Era of 1863-1914. In this period, there was no central bank to act as a regulator or a public backstop for the banking sector, and commercial banks were subject to frequent bouts of early withdrawals (Gorton and Tallman, 2016). Another potential application of this framework is emerging market economies in contemporary times.

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Appendix

A Appendix of Chapter 1

A1 Risk aversion

Under risk aversion, the deposit demand schedule is given by

$$q(\gamma, d) = \begin{cases} q^* & \text{for } d \leq \bar{d}(\gamma) \\ q^* \frac{1-P+P\frac{u_c(\underline{c})}{u_c(c)}\left(\gamma\frac{\theta^b}{q^b}+(1-\gamma)\frac{\theta^l}{q^l}\right)\frac{n}{d}}{1-q^* P\frac{u_c(\underline{c})}{u_c(c)}\left(\gamma\frac{\theta^b}{q^b}+(1-\gamma)\frac{\theta^l}{q^l}\right)} & \text{for } d > \bar{d}(\gamma) \end{cases}$$

where $u_c(.)$ is marginal utility and $(\underline{c}, c)$ are respectively consumption in states with strong and weak fundamentals. The marginal utility wedge $\frac{u_c(\underline{c})}{u_c(c)}$ exceeds unity and increases in d. Compared to the case with risk neutrality, this leads to a small discontinuity in $q(\gamma, d)$ around the deposit threshold and increases the curvature of the schedule in the risky region $d > \bar{d}(\gamma)$. This in turn raises the mark-up

$$\mu_d(\tilde{\gamma}, d) \equiv -\frac{\partial q(\tilde{\gamma}, d)}{\partial d}\frac{d}{q}$$

such that there is an interior solution $\gamma_g \in (0, 1)$ under the gambling strategy.

A2 Deviation to the safe strategy

In the first case, the bank has sufficient net worth to satisfy the first order condition (1.26) while remaining within the deposit threshold $\bar{d}(\gamma_g)$. Its deposits are thus valued on par with safe assets $q_{s|g} = q^*$ and its valuation of loans is equivalent to the equilibrium counterpart such that $q^l_{s|g} = q^l_s$. There are, however, two notable differences. First, as with the deviation to gambling, the quantity of lending $l_{s|g}$ is conditional on lending provided by the remaining banks such that

$$l_{s|g} = \left(q^l_s\right)^{\frac{\alpha}{1-\alpha}} (\alpha A)^{\frac{1}{1-\alpha}} - \frac{1-\phi}{\phi} l_g \tag{A33}$$

Second, the inward shift in the deposit threshold $\bar{d}(\gamma_g)$ under bad sentiments increases the boundary of net worth

$$n_{r|g} \equiv \left(\frac{q^b - \theta^b q^*}{q^b}\right) q^l_s l_{s|g} > n_c \tag{A34}$$

required for this case to be valid.

Note also that the deviating bank's expected payoff is given by the expression

$$v_{s|g} = \left(1 - P + P\theta^l\right) \mu_l l_{s|g} + \frac{n}{q^*} \tag{A35}$$

which differs from (1.39) only in terms of $l_{s|g}$.[99] This reflects that a shift to bad sentiments has no impact on the bank's ability to borrow when its net worth lies above $n_{r|g}$.

In the second case, bank net worth falls short of $n_{r|g}$ such that it is not possible to satisfy (1.26) without breaching the deposit threshold $\bar{d}\left(\gamma_g\right)$. The optimal allocation leaves the bank with a level of deposits $d_{s|g} > \bar{d}\left(\gamma_g\right)$ which is in the "risky" region of the deposit demand schedule with $q_{s|g} < q^*$, while the actual solvency constraint is slack. Proposition 1.2 indicates that there are no risky asset purchases $\left(b_{s|g} = 0\right)$ in this case, while the price and quantity of loans are pinned down by the first order condition (1.26) as

$$\begin{aligned} q^l_{s|g} &= \left(1 - P + P\theta^l\right)\left(1 - \mu_l\right) q^b && \text{(A36)} \\ l_{s|g} &= \left(q^l_{s|g}\right)^{\frac{\alpha}{1-\alpha}} \left(\alpha A\right)^{\frac{1}{1-\alpha}} - \frac{1-\phi}{\phi} l_g && \text{(A37)} \end{aligned}$$

Using the budget constraint, the price of deposits can also be written as

$$d_{s|g} = \frac{q^l_{s|g}}{q^b} l_{s|g} - \frac{n}{\left(1 - P\right) q^*}$$

and the deviating bank's expected payoff is given by the expression

$$v_{s|g} = \left(1 - P + P\theta^l\right) \mu_l l_{s|g} + \frac{n}{\left(1 - P\right) q^*} \tag{A38}$$

which is lower than (A35) due to the increase in bank funding costs.

The solvency constraint binds in the third case. The quantity of loans is determined implicitly by the expression

$$\left(\frac{\left(l_{s|g} + \frac{1-\phi}{\phi} l_g\right)^{\frac{1}{1-\alpha}}}{\left(\alpha A\right)^{\frac{1}{a}}} - q^b \theta^l \right) l_{s|g} = \frac{q^b}{1 - P} \frac{n}{q^*}$$

attained by using (1.47) and (A37) to substitute for $q\left(\gamma_g, d_{s|g}\right)$ and $q^l_{s|g}$ in (1.48). The expression for expected payoff in this case is identical to the constrained case of the safe equilibrium

$$v_{s|g} = \left(1 - P\right)\left(1 - \theta^l\right) l_{s|g} \tag{A39}$$

[99] As with the safe equilibrium, risky asset purchases $b_{s|g}$ and deposits $d_{s|g}$ are indeterminate in this case but have no impact on expected payoff.

and this case is valid when net worth is below the boundary

$$n < n_{c|g} \equiv \left(\frac{q^l_{s|g}}{q^b} - \theta^l \right) (1 - P) \, q^* l_{s|g} \tag{A40}$$

where $\left(q^l_{s|g}, l_{s|g}\right)$ are defined according to (A36) and (A37).

Finally, the discontinuous jump in $\mu_d\left(\gamma_g, d\right)$ as deposits $d_{s|g}$ cross the threshold $d\left(\gamma_g\right)$ leads to the possibility of a fourth case. In this case, net worth is below $n_{r|g}$ but the first order condition (A36) associated with the second case leads the bank to select a level of deposits within the threshold $d_{s|g} \leq d\left(\gamma_g\right)$. The optimal behaviour of the deviating bank, and the associated net worth boundaries can be then be determined by treating the deposit threshold as a binding constraint such that

$$\begin{aligned} d_{s|g} &= \bar{d}\left(\gamma_s\right) \\ q_{s|g} &= q^* \end{aligned}$$

and the level of deposits is determined implicitly by the expression

$$\frac{\left(l_{s|g} + \frac{1-\phi}{\phi} l_g\right)^{\frac{1}{1-\alpha}}}{\left(\alpha A\right)^{\frac{1}{a}}} l_{s|g} = \frac{\theta^b}{q^b - \theta^b q^*} n$$

The deviating bank's expected payoff is then given by

$$v_{s|g} = \left(1 - P + P\theta^l\right) l_{s|g} - \frac{\theta^b}{q^b - \theta^b q^*} n$$

This case is valid in the region of net worth $n \in [n_{cs|g}, n_{r|g})$ with $n_{cs|g}$ defined as

$$n_{cs|g} \equiv \frac{\left(1 - \theta^l\right)\left(1 - P\right)}{1 - P + P\theta^l} q^l_{s|g} l_{s|g}$$

A3 Proofs of propositions and lemmata

A3.1 Proof of Lemma 1.1

Suppose there is perfect transparency and limited liability binds such that the representative bank's optimization problem is given by

$$\begin{aligned} v &= \max_{d,b,l} (1-P)(b+l-d) \\ & \text{s.t.} \\ n+qd &= q^b b + q^l l \\ q^l &= \left(\frac{1}{\alpha A}\right)^{\frac{1}{\alpha}} (l+(1-\phi)L)^{\frac{1-\alpha}{\alpha}} \\ q &= q^* \left(1-P+P\frac{\theta^b b+\theta^l l}{d}\right) \end{aligned}$$

Combining the deposit demand schedule and the budget constraint yields

$$d = \frac{q^b b + q^l l - n - Pq^*\left(\theta^b b + \theta^l l\right)}{q^*(1-P)}$$

By using this expression to substitute out d and applying positive monotonic transformations, the objective function can be written as

$$\begin{aligned} v &= \max_{b,l} \left[q^*\left(1-P+P\theta^b\right) - q^b\right] b + \left[q^*\left(1-P+P\theta^l\right) - q^l\right] l + n \\ & \text{s.t.} \\ q^l &= \left(\frac{1}{\alpha A}\right)^{\frac{1}{\alpha}} (l+(1-\phi)L)^{\frac{1-\alpha}{\alpha}} \end{aligned}$$

which is identical to the optimization problem under a safe strategy.

A3.2 Proof of Lemma 1.2

Combining (1.25) with (1.1) yields

$$\left(1-P+P\theta^b\right)\left(\left(1-\mu_d(\tilde{\gamma},d)\right)q - q^*\right) \leq \lambda\left(\left(1-P+P\theta^b\right)q^* - \theta^b\left(1-\mu_d(\tilde{\gamma},d)\right)q\right) \quad \text{(A41)}$$

When $\lambda = 0$ and $q = q^*$, this reduces to

$$q^b = \left(1-P+P\theta^b\right)q^*$$

In contrast, when $q > q^*$, it is easy to show with the use of (1.18) that the LHS of (A41) is strictly negative under the condition

$$\left(\gamma\frac{\theta^b}{q^b} + (1-\gamma)\frac{\theta^l}{q^l}\right) < \frac{1}{q^*}$$

which is always satisfied under the parameter restrictions (1.15). Similarly, when $\lambda > 0$, the RHS is strictly positive under the condition

$$\theta^b < 1$$

when $q = q^*$. When $q < q^*$, the condition instead becomes

$$(1-P)\left(1-\theta^b\right) + \frac{1 - q^*\left(\gamma\frac{\theta^b}{q^b} + (1-\gamma)\frac{\theta^l}{q^l}\right)}{1 - q^*P\left(\gamma\frac{\theta^b}{q^b} + (1-\gamma)\frac{\theta^l}{q^l}\right)} P\theta^b > 0$$

which is again satisfied at all times under the parameter restrictions (1.15). As such, (1.25) holds with a strict inequality when either $\lambda > 0$ or $q < q^*$. In this case, the bank's valuation of risky assets is below their market price and the bank optimally decides not to purchase any such that

$$b = 0$$

A3.3 Proof of Lemma 1.3

Since the RHS of (1.40) increases in n and the only variable that is not pinned down in the RHS is l_s, l_s will be positively related to n if the LHS is increasing in l_s. If we define the left hand side as

$$f(l_s) \equiv \left(\frac{1}{\phi}\right)^{\frac{1-\alpha}{\alpha}} \left(\frac{l_s}{\alpha A}\right)^{\frac{1}{\alpha}} - q^*\theta^l l_s$$

the derivative with respect to l_s is given by

$$f'(l_s) = \frac{1}{\alpha}\left(\frac{1}{\alpha A}\right)^{\frac{1}{\alpha}} \left(\frac{1}{\phi}\right)^{\frac{1-\alpha}{\alpha}} l_s^{\frac{1-\alpha}{a}} - q^*\theta^l$$

and is positive for the range of l_s

$$l_s > \phi(\alpha)^{\frac{1+\alpha}{1-\alpha}} \left(A\left(q^*\theta^l\right)^{\alpha}\right)^{\frac{1}{1-\alpha}}$$

Using (1.40), it is also possible to attain an explicit expression for l_s when $n = 0$. This yields

$$l_s = \phi \left(A \left(q^* \theta^l \right)^{\alpha} \right)^{\frac{1}{1-\alpha}}$$

which satisfies the condition immediately above since $0 < \alpha < 1$.

A3.4 Proof of Lemma 1.4

When the solvency constraint is slack such that $n \geq n_c$, combining (1.43) with (1.39) and (1.46) yields

$$\begin{aligned} v_s &\geq v_{g|s} \\ \rightarrow 1 + \phi \frac{P}{1-P} \theta^l &\geq \left(\frac{1 - P + P\theta^b}{1 - P + P\theta^l} \right)^{\frac{\alpha}{1-\alpha}} \end{aligned}$$

As θ^l rises, LHS increases and RHS falls so a sufficient condition is attained by setting θ^l to its lower bound $(1 - \mu_l)\, \theta^b$

$$1 + \phi \left(1 - \mu_l\right) \frac{P}{1-P} \theta^b \geq \left(\frac{1 - P + P\theta^b}{1 - P + P\left(1 - \mu_l\right) \theta^b} \right)^{\frac{\alpha}{1-\alpha}}$$

This condition holds with equality when $\phi = 0$. To show that it is satisfied for all $\phi > 0$, consider the condition for the derivative of the LHS with respect to ϕ to exceed that of the RHS

$$1 - P + \frac{\alpha}{\alpha + \phi\left(1 - \alpha\right)} P\theta^b > \left(1 - P\right)^{1-\alpha} \left(1 - P + P\theta^b\right)^{\alpha}$$

which is satisfied for all $\phi \in (0, 1]$ when $\alpha \in (0, 1)$.where $\left(q^l_{s|g}, l_{s|g} \right)$ are given by (A36) and (A37).

A3.5 Proof of Proposition 1.1

Using (1.30) and (1.35), it is straight forward to show that

$$q^l_s < q^l_g \leftrightarrow \theta^l < \theta^b$$

where the right hand side reflects the unconstrained case. As (1.31) and (1.36) establish a monotonic relationship between the price of loans q^l and the amount of lending l in both equilibria, this also implies that $q^l_g l_g < q^l_s l_s$ when the solvency constraint is slack under the safe strategy. When the solvency constraint is binding, on the other hand, (1.40) can be used to determine the level of net worth above which $l_g \geq l_s$ as

$$n \geq \left(\frac{1}{\phi}\right)^{\frac{1-\alpha}{\alpha}} \left(\frac{l_g}{\alpha A}\right)^{\frac{1}{\alpha}} - q^* \theta^l l_g$$

where $q_g^l l_g < q_s^l l_s$ again follows from monotonicity.

A3.6 Proof of Proposition 1.2

Proposition 1.4 shows that a safe equilibrium always exists when $n \geq n_c$ such that the boundary $\underline{n}$ for a unique gambling equilibrium must lie within the region of net worth where the solvency constraint is binding under the safe equilibrium. The condition to eliminate the safe equilibrium can then be attained by combining (1.43) with (1.42) and (1.46) such that

$$(1-P)\left[\left(1-\theta^l\right)+\mu_l \frac{1-\phi}{\phi}\right] l_s < (1-P)\,\mu_l \left(q_g^l\right)^{\frac{\alpha}{1-\alpha}} (\alpha A)^{\frac{1}{1-\alpha}} + \frac{n}{q^*}$$

where l_s is given by (1.40). Proposition 1.3 then indicates that the safe equilibrium ceases to be self-confirming when $n < \underline{n}$ where combining (1.40) with the above condition yields an implicit expression for the boundary

$$\begin{aligned} \underline{n} \;=\; & \left(\frac{1}{\phi}\right)^{\frac{1-\alpha}{\alpha}} \left(\frac{1}{A\alpha} \frac{q^*(1-P)\,\mu_l \left(q_g^l\right)^{\frac{\alpha}{1-\alpha}} (A\alpha)^{\frac{1}{1-\alpha}} + \underline{n}}{q^*(1-P)\left[1-\theta^l+\mu_l \frac{1-\phi}{\phi}\right]}\right)^{\frac{1}{\alpha}} \\ & -q^*\theta^l \frac{q^*(1-P)\,\mu_l \left(q_g^l\right)^{\frac{\alpha}{1-\alpha}} (A\alpha)^{\frac{1}{1-\alpha}} + \underline{n}}{q^*(1-P)\left[\left(1-\theta^l\right)+\mu_l \frac{1-\phi}{\phi}\right]} \end{aligned} \qquad \text{(A42)}$$

Second, consider the condition for the gambling equilibrium to be self-confirming. When $n \geq n_{r|g}$ such the deposits are priced at the risk-free level $q_{s|g} = q^*$ under a deviation to the safe strategy, the relevant condition is attained by combining (1.44) with (1.17) and (A35) such that the gambling equilibrium is not self-confirming when

$$\begin{aligned} & v_g < v_{s|g} \\ \rightarrow 1 - \frac{\phi P \theta^l}{1-P+P\theta^l} & < \left(\frac{1-P+P\theta^l}{1-P+P\theta^b}\right)^{\frac{\alpha}{1-\alpha}} \end{aligned}$$

As θ^l rises, LHS decreases and RHS falls so a sufficient condition is attained by setting θ^l to its lower bound $(1-\mu_l)\,\theta^b$

$$1-\frac{\phi\,(1-\mu_l)\,P\theta^l}{1-P+P\,(1-\mu_l)\,\theta^l}<\left(1-\frac{\mu_l P\theta^l}{1-P+P\theta^l}\right)^{\frac{\alpha}{1-\alpha}}$$

The two sides of the inequality are equal when $\phi=0$. To show that the condition satisfied for all $\phi>0$, consider the condition for the derivative of LHS to be lower than that of RHS for all $\phi\in(0,1]$

$$\begin{aligned}&\frac{1}{1-P+P\,(1-\mu_l)\,\theta^l}\left[\frac{1-\alpha}{\alpha}\frac{(1-P)\,(1-\mu_l)\,\phi}{\left(1-P+P\,(1-\mu_l)\,\theta^b\right)}-1\right]\\ <\;&\frac{1-\mu_l}{1-P+P\theta^b}\left(1-\frac{\mu_l P\theta^b}{1-P+P\theta^b}\right)^{-\frac{1-2\alpha}{1-\alpha}}\end{aligned}$$

which is true since the LHS is negative and the RHS is positive. As such, the gambling equilibrium is not self-confirming when net worth is above $n\geq n_{r|g}$. Next, consider the case when net worth is in the range $n\in[n_{c|g},n_{r|g})$ such that the deposits of the deviating bank are priced at "risky" region of the deposit threshold $q_{s|g}<q^*$ but the solvency constraint remains slack. The condition for the gambling equilibrium to be self-confirming in this case can be attained by combining (1.44) with (1.17) and (A38) such that

$$\left(1-P+P\theta^l\right)\mu_l l_{s|g}+\frac{n}{(1-P)\,q^*}<(1-P)\,\mu_l l_g+\frac{n}{q^*}$$

where $l_{s|g}$ is defined according to (A33). This can be re-written as a requirement that net worth is below the boundary

$$\bar{n}\equiv\frac{(1-P)\,q^*}{P}\left[(1-P)+P\theta^l\,(1-\phi)-\left(1-P+P\theta^l\right)^{\frac{1}{1-\alpha}}\right]\left((1-\mu_l)\,q^b\right)^{\frac{\alpha}{1-\alpha}}(\alpha A)^{\frac{1}{1-\alpha}}\mu_l$$

Finally, it is necessary to show that $\bar{n}>n_{c|g}$ such that the boundary lies in the region where the solvency constraint is slack. Using the above expression and (A40), the relevant condition can be written as

$$\left(\frac{1}{1-P+P\theta^l}\right)^{\frac{\alpha}{1-\alpha}}>1+\frac{\phi P\alpha\left(1-\theta^l\right)}{\phi\,(1-\alpha)+P\alpha\left(1-\theta^l\right)(1-\phi)}\tag{A43}$$

Both sides of the inequality are decreasing in θ^l, but the LHS decreases faster when

$$\left(\frac{1}{1-P+P\theta^l}\right)^{\frac{1}{1-\alpha}}>\left(\frac{\phi\,(1-\alpha)}{\phi\left(1-\alpha\left(1-P+P\theta^l\right)\right)+P\alpha\left(1-\theta^l\right)}\right)^2$$

which is true for all $\phi \in (0,1]$. A sufficient condition can then be attained by substituting the upper bound $\theta^l = \frac{(1-P)(1-\mu_l)}{1-P+P\mu_l}$ into (A43)

$$\left(1+\frac{P}{1-P}\mu_l\right)^{\frac{\alpha}{1-\alpha}} > 1+\frac{\phi P(1-\mu_l)}{1-\phi P(1-\mu_l)}$$

where the two sides are equal when $\phi = 0$. The inequality is satisfied for all $\phi \in (0,1]$ when the LHS rises faster than RHS, which is true under the sufficient condition

$$\begin{aligned} 0 &< \alpha \leq \frac{1}{2} \\ 0 &< \phi \leq \frac{1}{2} \end{aligned}$$

A3.7 Proof of Proposition 1.3

First, consider the condition for the non-emptiness of the region $n \in (0, \underline{n}]$ with a unique gambling. When $n = 0$, (1.40) yields an explicit solution for bank lending

$$l_e = \phi(\alpha A)^{\frac{1}{1-\alpha}}\left(q^*\theta^l\right)^{\frac{\alpha}{1-\alpha}}$$

and using (1.43), (1.42) and (1.46), the condition to eliminate the safe equilibrium can be written as

$$\frac{\phi}{\mu_l}\left(1-\theta^l\right)+1-\phi < \left(\frac{(1-\mu_l)\left(1-P+P\theta^b\right)}{\theta^l}\right)^{\frac{\alpha}{1-\alpha}}$$

In the remainder of the proof, I show that there is a boundary $\underline{\theta}^l$ within the parameter restrictions given by (1.15) such that the safe equilibrium is not self-confirming for $\theta^l < \underline{\theta}^l$. The above expression can be used to define the function

$$f\left(\theta^l\right) \equiv \left(\frac{(1-\mu_l)\left(1-P+P\theta^b\right)}{\theta^l}\right)^{\frac{\alpha}{1-\alpha}} - (1-\phi) - \frac{\phi}{\mu_l}\left(1-\theta^l\right) \tag{A44}$$

where $f\left(\theta^l\right) < 0$ confirms the safe equilibrium. The derivative

$$f'\left(\theta^l\right) = \frac{\phi}{\mu_l} - \frac{\alpha}{1-\alpha}\left((1-\mu_l)\left(1-P+P\theta^b\right)\right)^{\frac{\alpha}{1-\alpha}}\left(\theta^l\right)^{\frac{-1}{1-\alpha}}$$

is strictly decreasing when θ^l is in the region

$$(1-\mu_l)\,\theta^b < \theta^l < (1-\mu_l)\,(1-P+P\theta^s)^{\alpha} \tag{A45}$$

where the lower bound corresponds to that of the parameter restriction (1.15) and the upper bound lies strictly within its counterpart there. It then follows from the intermediate value theorem that there is a boundary $\underline{\theta}^l$ within (A45) and hence (1.15) where $f\left(\underline{\theta}^l\right) = 0$ such that the safe equilibrium ceases to be self-confirming for $\theta^l < \underline{\theta}^l$. This boundary is implicitly defined by equating (A44) to zero such that

$$(1-\phi) + \phi\frac{1-\underline{\theta}^l}{\mu_l} = \left(\frac{(1-\mu_l)\left(1-P+P\theta^b\right)}{\underline{\theta}^l}\right)^{\frac{\alpha}{1-\alpha}}$$

To confirm that there is a non-empty region with a unique gambling equilibrium, it is also necessary to show that $\bar{n} > 0$ such that there is a region where the gambling equilibrium is self-confirming. As the proof for Proposition 1.2 shows that $\bar{n} > n_{c|g}$, a sufficient condition is given by

$$\begin{gathered} n_{c|g} > 0 \\ \rightarrow \left(1-P+P\theta^l\right)(1-\mu_l) > \theta^l \end{gathered}$$

which is true when θ^l is below the upper bound of (1.15).

The second part of the proof establishes that there is a non-empty region with multiple equilibria such that $\bar{n} > \underline{n}$. When $\theta^l > \underline{\theta}^l$ such that there is no unique gambling equilibrium and $\underline{n} = 0$, this follows directly from the above finding that $\bar{n} > 0$. When a unique gambling equilibrium exists, on the other hand, the proof is more involved as $\underline{n}$ is only implicitly defined by (1.50). Using (1.50), let

$$\phi(n) \equiv \left(\frac{1}{\phi}\right)^{\frac{1-\alpha}{\alpha}} \left(\frac{1}{A\alpha}\frac{q^*(1-P)\mu_l\left(q_g^l\right)^{\frac{\alpha}{1-\alpha}}(A\alpha)^{\frac{1}{1-\alpha}}+n}{q^*(1-P)\left[1-\theta^l+\mu_l\frac{1-\phi}{\phi}\right]}\right)^{\frac{1}{\alpha}} - \theta^l\frac{q^*(1-P)\mu_l\left(q_g^l\right)^{\frac{\alpha}{1-\alpha}}(A\alpha)^{\frac{1}{1-\alpha}}+n}{(1-P)\left[\left(1-\theta^l\right)+\mu_l\frac{1-\phi}{\phi}\right]}$$

such that $\phi(\underline{n}) = 0$. It follows from Proposition 1.3 that $\phi(.)$ is strictly decreasing within the region $n \in [0, n_c]$ such that $\bar{n} > \underline{n}$ implies

$$\phi(\bar{n}) < \phi(\underline{n}) = 0$$

Using (1.50) and (1.51), this can be written as the condition

$$\frac{\mu_l}{\phi}\frac{1+\kappa}{\left(1-\theta^l\right)+\mu_l\frac{1-\phi}{\phi}} < \left[\frac{q^*}{q_g^l}\left(\left(1-\theta^l+\mu_l\frac{1-\phi}{\phi}\right)(1-P)\frac{\kappa}{1+\kappa}+\theta^l\right)\right]^{\frac{\alpha}{1-\alpha}} \tag{A46}$$

where I have defined

$$\kappa \equiv \frac{1}{P}\left[(1-P)+P\theta^{l}(1-\phi)-\left(1-P+P\theta^{l}\right)^{\frac{1}{1-\alpha}}\right]>0$$

and $\kappa > 0$ follows directly from the proof above for $\bar{n} > 0$. The LHS of (A46) is less than one for $\theta^l < 1$ such that a sufficient condition under which (A46) is satisfied is given by

$$\alpha\left(1-P+P\theta^{b}\right)<\left(1-(a+(1-\alpha)\phi)\theta^{l}\right)(1-P)\frac{\kappa}{1+\kappa}+\theta^{l}$$

Since $\kappa > 0$, the first term on the RHS can be eliminated to get a stricter sufficient condition

$$\theta^{l}>\alpha\left(1-P+P\theta^{b}\right)$$

The RHS lies below the lower bound $\theta^l = (1-\mu_l)\,\theta^b$ of the parameter restriction (1.15) when the following condition is true

$$\frac{\theta^{b}}{a+(1-\alpha)\phi}>1-P+P\theta^{b}$$

A3.8 Proof of Proposition 1.4

The gambling equilibrium becomes unique for all n when the expected payoff under a deviation to gambling exceeds the expected payoff from the strategy given a slack solvency constraint. The relevant inequality is

$$v_s < v_{g|s}$$

where $\left(v_s, v_{g|s}\right)$ are respectively given by (1.39) and

$$v_{g|s}=(1-P)\,\mu_l l_{g|s}+P\bar{d}_c$$

such that the inequality becomes

$$\bar{d}^{c}>\tilde{d}^{c}\equiv\frac{\mu_l}{P}\left[\left(1-P+P\theta^{l}\right)l_s-(1-P)\,\mu_l l_{g|s}\right]$$

The expression given in the proposition can then be attained by substituting for $\left(l_s, l_{g|s}, q_s^l, q_g^l\right)$ using (1.30), (1.35), (1.36), (1.45).

The second part largely follows from the beginning of Appendix A3.6. The counterpart to

(A42) under liquidity provision is

$$\begin{aligned}\underline{n} &= \left(\frac{1}{\phi}\right)^{\frac{1-\alpha}{\alpha}}\left(\frac{1}{A\alpha}\frac{q^*\left(1-P\right)\mu_l\left(q_g^l\right)^{\frac{\alpha}{1-\alpha}}\left(A\alpha\right)^{\frac{1}{1-\alpha}}+\underline{n}+q^*\bar{d}^c}{q^*\left(1-P\right)\left[1-\theta^l+\mu_l\frac{1-\phi}{\phi}\right]}\right)^{\frac{1}{\alpha}} \\ &\quad -\theta^l\frac{q^*\left(1-P\right)\mu_l\left(q_g^l\right)^{\frac{\alpha}{1-\alpha}}\left(A\alpha\right)^{\frac{1}{1-\alpha}}+\underline{n}+q^*\bar{d}^c}{\left(1-P\right)\left[\left(1-\theta^l\right)+\mu_l\frac{1-\phi}{\phi}\right]}\end{aligned}$$

where the derivative with respect to $\bar{d}^c$ is

$$\begin{aligned}\frac{\partial\underline{n}}{\partial\bar{d}^c} &= \frac{\bar{d}^c}{\left(1-P\right)\left[\left(1-\theta^l\right)+\mu_l\frac{1-\phi}{\phi}\right]}* \\ &\quad \left(\frac{1}{\alpha}\left(\frac{1}{\alpha A}\right)^{\frac{1}{\alpha}}\left(\frac{q^*\left(1-P\right)\mu_l\left(q_g^l\right)^{\frac{\alpha}{1-\alpha}}\left(aA\right)^{\frac{1}{1-a}}+\underline{n}+q^*L}{q^*\phi\left(1-P\right)\left[\left(1-\theta^l\right)+\mu_l\phi\right]}\right)^{\frac{1-\alpha}{\alpha}}-q^*\theta^l\right)\end{aligned}$$

which is positive for $\underline{n}\geq 0$.

A3.9 Proof of Proposition 1.5

Using (1.55) and (1.57), we can write

$$\begin{aligned}q^{liq}\left(\tilde{\gamma},d,x\right)d+q^*x &= q^*\frac{1-P+P\left(\tilde{\gamma}\frac{\theta^b}{q^b}+\left(1-\tilde{\gamma}\right)\frac{\theta^l}{q^l}\right)\left(\frac{n+q^*x}{d}\right)}{1-q^*P\left(\tilde{\gamma}\frac{\theta^b}{q^b}+\left(1-\tilde{\gamma}\right)\frac{\theta^l}{q^l}\right)}d+q^*x \\ &= q^*\frac{\left(1-P\right)d+P\left(\tilde{\gamma}\frac{\theta^b}{q^b}+\left(1-\tilde{\gamma}\right)\frac{\theta^l}{q^l}\right)n+x}{1-q^*P\left(\tilde{\gamma}\frac{\theta^b}{q^b}+\left(1-\tilde{\gamma}\right)\frac{\theta^l}{q^l}\right)} \\ &= q^F\left(\tilde{\gamma},d,x\right)d\end{aligned}$$

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B Appendix of Chapter 2

B1 Liquidity Provision

In periods $t \leq T$, the model is characterized by

$$\bar{d}(n,\mathbf{S}) = \frac{\left(\tilde{\gamma}(n,\mathbf{S})\frac{\theta^b}{q^b(\mathbf{S})} + (1-\tilde{\gamma}(n,\mathbf{S}))\frac{\theta^l}{q^l(\mathbf{S})}\right)(n+q^* d_t^c)}{1-q^*\left(\tilde{\gamma}(n,\mathbf{S})\frac{\theta^b}{q^b(\mathbf{S})} + (1-\tilde{\gamma}(n,\mathbf{S}))\frac{\theta^l}{q^l(\mathbf{S})}\right)}$$

$$q(d',n,\mathbf{S}) = \begin{cases} q^* & \text{for } d' \leq \bar{d}(n,\mathbf{S}) \\ q^*\frac{1-P(\mathbf{S})+P(\mathbf{S})\frac{u_c(\underline{c})}{u_c(c')}\left(\gamma_g\frac{\theta^b}{q^b(\mathbf{S})}+\left(1-\gamma_g\right)\frac{\theta^l}{q^l(\mathbf{S})}\right)\frac{n+q^*d_t^c}{d'}}{1-q^*P(\mathbf{S})\frac{u_c(\underline{c})}{u_c(c')}\left(\gamma_g\frac{\theta^b}{q^b(\mathbf{S})}+\left(1-\gamma_g\right)\frac{\theta^l}{q^l(\mathbf{S})}\right)} & \text{for } d' > \bar{d}(n,\mathbf{S}) \end{cases}$$

$$\pi = l + b - d' - d_t^c$$

$$\underline{\pi} = \max\left(\theta^l l + \theta^b b - d' - d_t^c, 0\right)$$

$$d' + d_t^c \leq \theta^l l + \theta^b b$$

$$v_t^b(n;\mathbf{S}) = \max\left\{v_{s,t}^b(n;\mathbf{S}), v_{g,t}^b(n;\mathbf{S})\right\}$$

$$v_{s,t}^b(n;\mathbf{S}) = \max_{d',d_t^c\leq\bar{d}_t^c,\gamma\in[0,1]}\left\{\begin{array}{c}(1-P(\mathbf{S}))\left((1-\psi)\pi + \psi E_{\mathbf{S}}\left[v_{t+1}^b(n';\mathbf{S}')\right]\right) \\ +P(\mathbf{S})\left((1-\psi)\underline{\pi} + \psi\underline{v}^b(\underline{n}';\mathbf{S})\right)\end{array}\right\}$$

$$v_{g,t}^b(n;\mathbf{S}) = \max_{d',d_t^c\leq\bar{d}_t^c,\gamma\in[0,1]}\left\{(1-P(\mathbf{S}))\left((1-\psi)\pi + \psi E_{\mathbf{S}}\left[v_{t+1}^b(n';\mathbf{S}')\right]\right)\right\}$$

B2 Proofs of propositions and lemmata

B2.1 Proof of Lemma 2.1

Let an underbar denote variables in the period immediately after sovereign default and the subscript 'ss' denote those that pertain to the steady state. The household's problem can then be written as

$$\begin{aligned}
\underline{v}^h(D,D^*;\mathbf{S}) &= \max_{\underline{c},\underline{D},\underline{D}^*}\left\{u(\underline{c}) + \beta v_{ss}^h(\underline{D},\underline{D}^*)\right\} \\
v_{ss}^h(\underline{D},\underline{D}^*) &= \max_{c_{ss},D_{ss},D_{ss}^*}\left\{u(c_{ss}) + \beta v_{ss}^h(D_{ss},D_{ss}^*)\right\} \\
&\quad \text{s.t.} \\
\underline{c} + \underline{q}\underline{D} + q^*\underline{D}^* &= \theta D + D^* - \underline{T} + \underline{w}(\mathbf{S}) \\
c_{ss} + q_{ss}D_{ss} + q^*D_{ss}^* &= \underline{D} + \underline{D}^* - \underline{T} + w_{ss} \\
c_{ss}' + q_{ss}D_{ss}' + q^*D_{ss}^{*\prime} &= D_{ss} + D_{ss}^* - \underline{T} + w_{ss}
\end{aligned}$$

where $\underline{w}(\mathbf{S})$ and w_{ss} are respectively given by

$$\begin{aligned} \underline{w}(\mathbf{S}) &= (1-\alpha)\underline{A}K \\ \underline{w} &= (1-\alpha)\underline{AK^{\alpha}} \end{aligned} \tag{B47}$$

and taxes remain constant after government default.

The first order conditions of this problem are

$$\begin{aligned} \underline{q}u_c(\underline{c}) &= \beta u_c(c_{ss}) \\ q^* u_c(\underline{c}) &= \beta u_c(c_{ss}) \\ q_{ss}u_c(c_{ss}) &= \beta u_c(c'_{ss}) \\ q^* u_c(c_{ss}) &= \beta u_c(c'_{ss}) \end{aligned}$$

which immediately indicates that $\underline{q} = q_{ss} = q^*$. Furthermore, under the parameterization $q^* = \beta$, the FOCs lead to complete consumption smoothing such that

$$\underline{c} = c_{ss} = c'_{ss}$$

Given that wages and taxes are constant in the steady state, this implies

$$\begin{aligned} \underline{c} &= c_{ss} = (1-q^*)(\underline{D} + \underline{D}^*) - \underline{T} + w_{ss} \\ v^h_{ss}(\underline{D}, \underline{D}^*) &= \frac{1}{1-\beta}u(\underline{c}) \\ \underline{v}^h(D, D^*; \mathbf{S}) &= \frac{\beta}{1-\beta}u(\underline{c}) \end{aligned}$$

Using the budget constraint immediately after default, I write $\underline{c}$ in terms of $(\theta D, D^*)$

$$\underline{c} = (1-q^*)(\theta D + D^* + \underline{w}(\mathbf{S})) + q^* w_{ss} - \underline{T}$$

and use (2.4), (2.6) and (2.7) to write

$$\underline{w}(\mathbf{S}) = \frac{1-\alpha}{\alpha}\frac{\underline{A}}{A}L(\mathbf{S})$$

and re-label w_{ss} as $\underline{w}$ with a slight abuse of notation.

B2.2 Proof of Lemma 2.2

When the bank remains solvent with profits $\underline{\pi}$ after sovereign default, it continues into the steady state with net worth

$$\underline{n} = \psi\left(\underline{\pi} - \omega\right)$$

as per (2.15)

Let 'ss' denote variables that pertain to the steady state. The bank's problem is solved by the first order condition (2.26) which implies profits

$$\pi_{ss} = \mu_l \phi \underline{L} + \frac{n}{q^*}$$

where $\underline{L}$ is given by (2.27). The parameterization for (ψ, ω) given in Section 2.3.6 implies

$$\begin{aligned} \pi_{ss} &= \underline{\pi} \\ n_{ss} &= \underline{n} \end{aligned}$$

The representative bank's value then remains constant and is given by

$$\begin{aligned} \underline{v}^b\left(\underline{n}; \mathbf{S}\right) &= \left(1 - \psi\right)\underline{\pi} + \psi \underline{v}^b\left(\underline{n}; \mathbf{S}\right) \\ &= \underline{\pi} \end{aligned}$$

B2.3 Proof of Proposition 2.1

Let $q\left(d', n, \mathbf{S}\right) = q^*$, $\lambda\left(n, \mathbf{S}\right) = 0$ for all $(n; \mathbf{S}) \notin \mathrm{G}$ and guess that the value function takes the form

$$\begin{aligned} v^b\left(n; \mathbf{S}\right) &= A\left(n; \mathbf{S}\right) + \frac{n}{q^*}, \\ A\left(\mathbf{S}\right) &= \left\{ \begin{array}{c} A_g\left(\mathbf{S}\right)\ \forall\left(n; \mathbf{S}\right) \in \mathfrak{g} \\ A_s\left(\mathbf{S}\right)\ \forall\left(n; \mathbf{S}\right) \notin \mathfrak{g} \end{array} \right\} \end{aligned}$$

where $A\left(\mathbf{S}\right)$ does not depend on n since combining the guess with (2.17) implies

$$A\left(\mathbf{S}\right) = \max\left\{A_g\left(\mathbf{S}\right), A_s\left(\mathbf{S}\right)\right\}$$

For $q(d', n, \mathbf{S}) = q^*$, $\lambda(n, \mathbf{S}) = 0$, the first order conditions (2.21) and (2.22) under the safe strategy reduce to

$$\begin{aligned} q^b(\mathbf{S}) &= \left(1 - P(\mathbf{S}) + P(\mathbf{S})\,\theta^b\right) q^* \\ q^l(l, \mathbf{S}) &= (1 - \mu_l)\left(1 - P(\mathbf{S}) + P(\mathbf{S})\,\theta^l\right) q^* \end{aligned}$$

Combining these with the definitions for $(\pi, \underline{\pi})$ implies an expected profit

$$(1 - P(\mathbf{S}))\,\pi + P(\mathbf{S})\,\underline{\pi} = \left(1 - P(\mathbf{S}) + P(\mathbf{S})\,\theta^l\right)\mu_l \phi L(\mathbf{S}) + \frac{n}{q^*}$$

under the safe strategy. Using the above guess to characterize $E_{\mathbf{S}}\left[v^b(n'; \mathbf{S}')\right]$ in (2.17) yields

$$\begin{aligned} v_s^b(n; \mathbf{S}) &= (1 - P(\mathbf{S}))\,\pi + P(\mathbf{S})\,\underline{\pi} + (1 - P(\mathbf{S}))\left(\psi \mathbb{E}_{\mathbf{S}}[A(\mathbf{S}')] - \psi\omega\right) \\ &= \left(1 - P(\mathbf{S}) + P(\mathbf{S})\,\theta^l\right)\mu_l \nu L(\mathbf{S}) + (1 - P(\mathbf{S}))\,\psi\left(\mathbb{E}_{\mathbf{S}}[A(\mathbf{S}')] - \omega\right) + \frac{n}{q^*} \end{aligned}$$

The first order conditions (2.23) and (2.24) under the gambling strategy imply the expected profit

$$(1 - P(\mathbf{S}))\,\pi = (1 - P(\mathbf{S}))\,\mu_l \phi L(\mathbf{S}) + \frac{n}{q^*}$$

Using the guess to substitute for $E_s\left[v^b(n'; \mathbf{S}')\right]$ in (2.17) yields

$$\begin{aligned} v_g^b(n; \mathbf{S}) &= (1 - P(\mathbf{S}))\left(\pi + \psi\left(\mathbb{E}_S[A(\mathbf{S}')] - \omega\right)\right) \\ &= (1 - P(\mathbf{S}))\left(\mu_l \phi L(\mathbf{S}) + \psi\left(\mathbb{E}_{\mathbf{S}}[A(\mathbf{S}')] - \omega\right)\right) + \frac{n}{q^*} \end{aligned}$$

Matching coefficients implies the following definitions for $(A_g(\mathbf{S}), A_s(\mathbf{S}))$

$$\begin{aligned} A_s(\mathbf{S}) &= \left(1 - P(\mathbf{S}) + P(\mathbf{S})\,\theta^l\right)\mu_l \phi L(\mathbf{S}) + (1 - P(\mathbf{S}))\,\psi\left(\mathbb{E}_{\mathbf{S}}[A(\mathbf{S}')] - \omega\right) \\ A_g(\mathbf{S}) &= (1 - P(\mathbf{S}))\left(\mu_l \phi L(\mathbf{S}) + \psi\left(\mathbb{E}_{\mathbf{S}}[A(\mathbf{S}')] - \omega\right)\right) \end{aligned}$$

which confirm the initial guess. This implies

$$\frac{\partial \mathbb{E}_{\mathbf{S}}\left[v^b(n', \mathbf{S}')\right]}{\partial n'} = \frac{1}{q^*}$$

under the conditions stated above. Using $\psi = q^*$ and (2.14), it is easy to show that

$$\frac{\partial \mathbb{E}_{\mathbf{S}}\left[v^b(n', \mathbf{S}')\right]}{\partial \pi'} = \frac{\psi}{q^*} = 1$$

It is straightforward to deduce that this derivative exceeds unity when the above conditions are not satisfied. When the solvency constraint binds, a marginal increase in future net worth n' leads to a relaxation of the solvency constraint and reduces the associated multiplier $\lambda(n', \mathbf{S}')$. Similarly, when $q(d', n, \mathbf{S}) < q^*$, the bank can use the additional net worth to reduce its deposits by $1/q(d', n, \mathbf{S}) > 1/q^*$. In both cases, there is a rise in $\mathbb{E}_{\mathbf{S}}\left[v^b(n', \mathbf{S}')\right]$ in addition to the direct impact shown above.

B3 Data and calibration

B3.1 Sovereign default risk

Monthly data on the yields of German and Portuguese government bonds with a remaining maturity of 3 months is obtained from Bloomberg. The specific maturity is selected in order to extract quarterly sovereign default probabilities in line with the calibration of the model at a quarterly frequency. A time series for sovereign default probabilities $\hat{P}_t$ is extracted using the calibration for θ^b and (2.1). This is then converted to a series $\hat{\delta}_t$ of fiscal stress realizations through (2.2).

In order to calibrate the fiscal stress parameters $(\delta_{ss}, \rho_\delta, \sigma_\delta^2)$, I run the following OLS regression

$$\hat{\delta}_t = \beta_0 + \beta_1 \hat{\delta}_{t-1} + \varepsilon_t$$

which fits the AR(1) process given by (2.3) to $\hat{\delta}_t$. Note that the regression is conducted at monthly frequency so as to maximize the number of observations, which stands at 81. The mean and persistence of the monthly AR(1) process relate to the estimated coefficients as follows

$$\begin{aligned} \delta_{ss}^m &= \frac{\hat{\beta}_0}{1-\hat{\beta}_1} \\ \rho_\delta^m &= \hat{\beta}_1 \end{aligned}$$

while the variance is that of the residuals $(\sigma_\delta^m)^2 = var(\hat{\varepsilon}_t)$. It is straightforward to convert these into the parameters $(\delta_{ss}, \rho_\delta, \sigma_\delta^2)$ associated with a quarterly AR(1) process

$$\begin{aligned} \delta_{ss} &= \left(1 + \rho_\delta^m + (\rho_\delta^m)^2\right) \frac{1-\rho_\delta^m}{1-(\rho_\delta^m)^3} \delta_{ss}^m \\ \rho_{ss} &= (\rho_\delta^m)^3 \\ \sigma_\delta^2 &= \left(1 + \rho_\delta^m + (\rho_\delta^m)^2\right)(\sigma_\delta^m)^2 \end{aligned}$$

which yields the calibration for these parameters.

For the matching exercise in Section 2.4.4, $\hat{P}_t$ is converted to a quarterly frequency and then (2.2), (2.3) and the calibrated parameters $(\delta_{ss}, \rho_\delta, \sigma_\delta^2)$ are used to determine the series of sovereign risk shocks ε' needed to exactly match these probabilities. These probabilities are shown in the first panel of Figure 2.10.

B3.2 Loan interest rates

Data on loan interest rates is obtained from the 'Total MFI Interest Rate Statistics' database in ECB's Statistical Data Warehouse. It includes loans by Portuguese banks other than revolving loans and overdrafts, convenience and extended credit card debt with collateral and/or guarantees to counterparties which are classified as non-financial corporations. The maximum maturity of these loans is 3 months in line with the quarterly frequency of the model, and only loans classified as new business are included. The data is obtained at monthly frequency and converted to quarterly frequency by averaging.

B3.3 Bank funding costs

Deposit interest rates are used as a measure for bank funding costs. Deposit interest rate data is obtained from the 'Total MFI Interest Rate Statistics' database in the ECB's Statistical Data Warehouse. It includes deposits (new business) of all amounts from households and non-corporations with an agreed maturity of up to 1 year. The 1 year maturity is selected as it is the lowest maturity classification available. The data series is obtained at monthly frequency and converted to quarterly frequency by averaging.

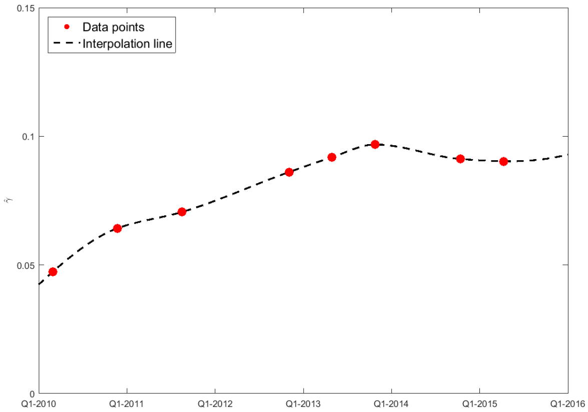
B3.4 Leverage ratio

Data on leverage ratios is obtained from the 'Consolidated Banking Data' database in the ECB's Statistical Data Warehouse. The data series provides the leverage ratio of domestic banking groups and stand-alone banks operating in Portugal and is available at semi-annual frequency. It is converted to quarterly frequency through interpolation.

B3.5 Domestic sovereign bond exposure

In order to construct the domestic sovereign bond exposure series $(\hat{\gamma}_t)$, I collect data on the sovereign bond holdings of Portuguese banks from stress tests conducted by the European Banking Authority and the ECB between 2009-2016. This data is at bank level and includes four Portuguese banks (Caixa Geral de Depositos, Banco Comercial Portuges, Espirito Santo Financial Group and Banco BPI) classified as 'systemically important' by the above institutions. Together, these banks account for approximately 60% of the total assets held by the Portuguese banking sector. I refer to this data series as $SOV_t\,(j)$ where j indexes the specific bank.

Figure B1 : **Domestic sovereign bond exposure**



Domestic sovereign bond exposure is defined as the share of bank funds spent on domestic sovereign bonds in Section 2.3.4. This requires data also on bank funding, which consists of liabilities and own funds. Accordingly, I compile data series for the total liabilities, $TL_t(j)$ and Tier 1 capital, $CAP_t(j)$ by going through the balance sheets of these individual banks.

The data series are aggregated across banks and then used to construct the domestic sovereign bond exposure according to the expression

$$\hat{\gamma}_t = \frac{\sum_j SOV_t(j)}{\sum_j (TL_t(j) + CAP_t(j))}$$

As there are only 8 observations through time provided by the stress tests, the constructed series is interpolated to create a quarterly series. Figure B1 provides a scatter plot of the data points on the interpolated line.

B4 Proofs for the numerical solution

In this section, I describe and prove two key lemmata which are used to reduce the computational burden.

Lemma B3 *The optimal choices under a safe strategy are independent of* $\left\{\Gamma(\mathbf{S}), X(\mathbf{S}), v^b(n, \mathbf{S})\right\}$ *when* $\tilde{\gamma}(n, \mathbf{S}) = \gamma_s$

Proof. The dependence on $X(\mathbf{S})$ is via the household's risk aversion affecting the deposit demand schedule. When good sentiments $\tilde{\gamma}(n, \mathbf{S}) = \gamma_s$ coincide with a safe strategy, the bank always stays on the safe region of the deposit demand schedule such its borrowing costs are at q^*. The deposit threshold itself does not depend on the marginal utility wedge such that $X(\mathbf{S})$ has no impact on the outcome.

The dependence on $\left\{\Gamma(\mathbf{S}), v^b(n,\mathbf{S})\right\}$, on the other hand, stems from the derivative term $\frac{\partial \mathbb{E}_{\mathbf{S}}\left[v^b(n',\mathbf{S}')\right]}{\partial \pi}$ in the FOCs associated with the safe strategy. To see why this is not an issue under $\tilde{\gamma}(n,\mathbf{S}) = \gamma_s$, consider the three alternative cases that the bank may be facing. First, consider the case where the solvency constraint is slack. The first order conditions from the safe strategy must then be

$$\begin{aligned}
\frac{q^b(\mathbf{S})}{q^*} &= (1-P(\mathbf{S}))\left(1-\psi+\psi\frac{\partial \mathbb{E}_{\mathbf{S}}\left[v^b(n',\mathbf{S}')\right]}{\partial \pi}\right) + P(\mathbf{S})\,\theta^b \\
\frac{q^l(l,\mathbf{S})}{(1-\mu_l)\,q^*} &= (1-P(\mathbf{S}))\left(1-\psi+\psi\frac{\partial \mathbb{E}_{\mathbf{S}}\left[v^b(n',\mathbf{S}')\right]}{\partial \pi}\right) + P(\mathbf{S})\,\theta^l
\end{aligned}$$

Combining the first FOC with (2.1) and using the parameterization $\psi = q^*$ yields

$$\frac{\partial \mathbb{E}_{\mathbf{S}}\left[v^b(n',\mathbf{S}')\right]}{\partial \pi} = 1$$

Therefore, this case is only valid under the conditions listed in Proposition 2.1. With the derivative term equal to unity, the FOCs become

$$\begin{aligned}
\frac{q^b(\mathbf{S})}{q^*} &= (1-P(\mathbf{S})) + P(\mathbf{S})\,\theta^b \\
\frac{q^l(l,\mathbf{S})}{(1-\mu_l)\,q^*} &= 1-P(\mathbf{S}) + P(\mathbf{S})\,\theta^l
\end{aligned} \tag{B48}$$

leading to the same outcome as the safe equilibrium (with a slack solvency constraint) in the two period environment.

Second, consider a second case where

$$\frac{\partial \mathbb{E}_{\mathbf{S}}\left[v^b(n',\mathbf{S}')\right]}{\partial \pi} > 1$$

due to expectations of becoming constrained in the future. This drives banks to borrow the maximum amount they can while remaining solvent such that the solvency constraint binds with a positive amount of sovereign bonds purchases $b > 0$

$$d' = \theta^l l + \theta^b b$$

The first order condition for lending then become

$$q^l(l,\mathbf{S}) = \frac{1-\mu_l}{1-\theta^b}\left[q^b(\mathbf{S})\left(1-\theta^l\right) + q^*\left(\theta^l-\theta^b\right)\right]$$

Note that the derivative $\frac{\partial \mathbb{E}_{\mathbf{S}}\left[v^b(n',\mathbf{S}')\right]}{\partial \pi}$ is absent from the first order condition as the bank is already attempting to maximize its payoff in the state without sovereign default within the constraints imposed by solvency. Substituting in for $q^b(\mathbf{S})$ using (2.1) yields the same expression as (B48) such that the outcome under the second case is identical to that of the first case.

Finally, when net worth is below

$$n < \left[\left(1 - P(\mathbf{S}) + P(\mathbf{S})\theta^l\right)(1-\mu_l) - \theta^l\right] q^* l_s$$

where l_s is consistent with (B48), the solvency constraint binds even in the absence of sovereign bond purchases such that $\lambda(n,\mathbf{S}) > 0$ and the outcome is identical to the safe equilibrium with a binding solvency constraint in the two period model. The derivative term has no impact here as the bank does not make any active choices other than borrowing as much as it can and investing it in loans, which is given by the implicit expression

$$q^l(l,\mathbf{S})\,l - q^*\theta^l l = n$$

Therefore, in all cases associated with a safe strategy and $\tilde{\gamma}(n,\mathbf{S}) = \gamma_s$, the outcome is independent of $\left\{\Gamma(\mathbf{S}), X(\mathbf{S}), v^b(n,\mathbf{S})\right\}$. ■

Lemma B4 *The optimal choices under a safe strategy are independent of* $\left\{\Gamma(\mathbf{S}), v^b(n,\mathbf{S})\right\}$ *when* $\lambda(n,\mathbf{S}) > 0$

Proof. For the case with good sentiments $\tilde{\gamma}(n,\mathbf{S}) = \gamma_s$, the final part of the proof for Lemma B3 shows that the outcome is independent of $\left\{\Gamma(\mathbf{S}), X(\mathbf{S}), v^b(n,\mathbf{S})\right\}$. Under bad sentiments $\tilde{\gamma}(n,\mathbf{S}) = \gamma_g$, $\lambda(n,\mathbf{S}) > 0$ again indicates that the bank does not make any active choices other than borrowing as much as it can and investing it in loans. The amount of lending is given implicitly by the expression

$$\left(q^l(l,\mathbf{S}) - q(d',n,\mathbf{S})\theta^l\right) l = n$$

where the deposit demand schedule $q(d',n,\mathbf{S})$ depends on the household's policy function $X(\mathbf{S})$. Therefore, the outcome is independent of $\left\{\Gamma(\mathbf{S}), v^b(n,\mathbf{S})\right\}$. ■

dBB

C Appendix of Chapter 3

C1 Proof of Proposition 3.1

First, we take the early withdrawal of shadow bank deposits as given and derive the no-withdrawal condition. Expected consumption is given by

$$(1-P)\,c_{bh} + pc_{bl} = M_1 + \theta^{SB} d^{SB} + \left(1 - p\left(1-\bar{V}\right)\right) d^{TB} R^{TB}$$

when households do not withdraw their deposits early from traditional banks, and

$$c_b^w = M_1 + \theta^{SB} d^{SB} + d^{TB}$$

when they do.[100] It is optimal for households not to withdraw their deposits when

$$\begin{gathered}(1-P)\,c_{bh} + pc_{bl} \geq c_b^w \\ \therefore V \geq \bar{V} \equiv \frac{1}{p}\left(\frac{1}{R^{TB}} - (1-p)\right)\end{gathered}$$

Jointly solving this with (3.13) yields $\bar{V} = 1$, $R^{TB} = 1$.

Next, we confirm that the early withdrawal from shadow banks is optimal. As shadow banks do not pay the commitment cost, they cannot commit to satisfying a no-withdrawal constraint. In equilibrium, it is optimal for households to withdraw their deposits early from shadow banks when

$$(1-p)\,R^{SB} + pV^{SB} R^{SB} < 1$$

where V^{SB} is defined according to (3.20). Using (3.14), (C51) and (C52), we can write

$$\begin{aligned} V^{SB} &= (1+\mu)\,\frac{\sigma_l}{\sigma_h} \\ R^{SB} &= \frac{\sigma_h}{(1-q)\,\sigma_h + q\,(1+\mu)\,P_2} \end{aligned}$$

and the condition becomes

$$\frac{(1-p)\,\sigma_h + p\,(1+\mu)\,\sigma_l}{(1-q)\,\sigma_h + q\,(1+\mu)\,P_2} < 1$$

Consider the condition when $p = q$. It will then be satisfied for any $P_2 > \sigma_l$ which must be true under (3.6). The necessary restriction on p depends on the mark-up μ. Specifically, when $\sigma_h < (1+\mu)\,\sigma_l$ we will need the restriction $p \geq q$ and vice versa with $\sigma_h < (1+\mu)\,\sigma_l$.

[100] Since households are atomistic, they do not internalize that their decision to withdraw deposits reduces the liquidation value of traditional banks.

C2 Limited liability

Let bl refer to the state with low asset payoffs. When the no-withdrawal constraint binds, the minimum recovery rate is limited to $\bar{V} \leq 1$ so that limited liability must also bind in bl. Therefore, we consider a scenario where both the no-withdrawal constraint and limited liability are slack. Since banks are always solvent, they borrow at the risk-free rate $R = 1$ and profits in bl are given by

$$\Pi_{bl} = \sigma_l I_2 + M_2 - D$$

and the bank's problem to maximize expected profits

$$\max_{I_1, I_2, M_1, M_2} \quad (1-q)\left[\sigma_h I_1 + M_1\right] + q\left(\left[(1-p)\,\sigma_h + p\sigma_l\right] I_2 + M_2\right) - D$$

subject to (3.17) and (3.19). Since $P_2 \leq (1-p)\,\sigma_h + p\sigma_l$ given $\phi \leq 1$, assets are priced at or below their expected payoff after bad news. Therefore banks weakly prefer investing in I_2 over M_2 such that we can set

$$\begin{aligned} I_2 &= I_1 + \frac{M_1}{P_2} \\ M_2 &= 0 \end{aligned}$$

This reduces the bank's problem to

$$\max_{I_1, M_1} \; I_1 + \left(1 - q + \frac{q}{\phi}\right) M_1 - (P_1 I_1 + M_1)$$

and the first order conditions for (I_1, M_1) are respectively

$$I_1 : P_1 = \frac{1}{(1+\mu)} \tag{C49}$$

$$M_1 : \phi = 1 \tag{C50}$$

where the inequality in (C50) due to the possibility of a binding no-short-sale constraint. The bank's payoff in bl can then be written as

$$\Pi_{bl} = \left(\sigma_l - \frac{1}{1+\mu}\right) I_1 + \left(\frac{\sigma_l}{(1-p)\,\sigma_h + p\sigma_l} - 1\right) M_1$$

and limited liability binds under the conditions

$$\sigma_l - \frac{1}{1+\mu} < 0$$
$$\frac{\sigma_l}{(1-p)\,\sigma_h + p\sigma_l} - 1 < 0$$

which are satisfied under (3.4). Therefore, we prove by contradiction that it is not possible for limited liability to be slack when assets yield a low payoff.

C3 Proof of Lemma 3.1

The first order conditions to this problem indicate that shadow banks do not find it optimal to hold any cash $\left(M_1^{SB} = 0\right)$ when $R^{SB} > 1$.[101] Therefore, their liquidation value can be written as

$$\theta^{SB} = \frac{P_2}{P_1^{SB}} \tag{C51}$$

where P_1^{SB} is pinned down by the first order condition for the risky asset

$$P_1^{SB} = \frac{\sigma_h}{1+\mu}\frac{1}{R^{SB}} \tag{C52}$$

Combining (C52) and (3.14) yields

$$P_1^{SB} = (1-q)\,\frac{\sigma_h}{1+\mu} + qP_2 \tag{C53}$$

and by substituting this into (C51) we attain

$$\theta^{SB} = \frac{(1+\mu)\,P_2}{(1-q)\,\sigma_h + q\,(1+\mu)\,P_2} \tag{C54}$$

There will be a liquidity shortfall when $\theta^{SB} < 1$. Since θ^{SB} is increasing in $P_2 = \phi\left[(1-p)\,\sigma_h + p\sigma_l\right]$, setting $\phi = 1$ provides a sufficient condition for this. With some algebra, we can write this condition as

$$(1+\mu)\left[(1-p)\,\sigma_h + p\sigma_l\right] < \sigma_h$$

A further sufficient condition can be attained by setting the mark-up to its maximum value $\mu = \bar{\mu}$. The condition then becomes $\sigma_h > 1$ which must be true.

[101] There is a no-short-sale constraint $(I_1, M_1) \geqq 0$ which is only binding for cash.

To get an expression for interest rates, we combine (C54) with (3.14) such that

$$R^{SB} = \frac{1}{1 - q + q\frac{P_2}{\sigma_h}(1+\mu)}$$

and $R^{SB} > 1$ follows from $\theta^{SB} < 1$.

Finally, substituting (3.17) and (C52) into (3.23) gives an expression for the expected payoff

$$E\left[\Pi^{SB}\right] = (1-q)\frac{\mu}{1+\mu}\sigma_h I_1^{SB}$$

where I_1^{SB} is attained by combining (C53) with the asset supply schedule (3.3) such that

$$I_1^{SB} = (A\alpha^{\alpha})^{\frac{1}{1-\alpha}}\left((1-q)\frac{\sigma_h}{1+\mu} + qP_2\right)^{\frac{\alpha}{1-\alpha}}$$

Observe that I_1^{SB}, P_1^{SB} and $E\left[\Pi^{SB}\right]$ are all positive related to P_2.

C4 Proof of Lemma 3.2

After substituting for $\left(R^{TB}, \bar{V}\right)$ as per (3.16) and dropping the label 'TB' to simplify the exposition, the traditional bank's problem can be written in as

$$\Pi = (1-q)(\sigma_h I_1 + M_1) + q(1-p)(\sigma_h I_2 + M_2) - (1-qp)D \tag{C55}$$

$$\text{s.t.}$$

$$P_1 I_1 + M_1 = D$$

$$P_2 I_2 + M_2 = P_2 I_1 + M_1 \tag{C56}$$

$$\sigma_l I_2 + M_2 \geq D \tag{C57}$$

$$(I_1, I_2, M_1, M_2) \geqq 0$$

where the last line represents no-short-sale constraints. There are three alternative cases depending on whether the no-withdrawal and no-short-sale constraint on I_2 bind. We first describe the case in Lemma 3.2, and then consider the remaining cases and prove that they may not be valid under the restrictions $\mu \leq \bar{\mu}$, $\phi > \underline{\phi}$

C4.1 Case 1

Lemma 3.2 describes the case where the no-withdrawal constraint (C57) and the no-short-sale constraint on I_2 bind. With $I_2 = 0$, the second period budget constraint (C56) and the no-withdrawal constraint can respectively be written as

$$
\begin{aligned}
M_2 &= P_2 I_1 + M_1 \\
M_2 &= P_1 I_1 + M_1
\end{aligned}
$$

Therefore, the no-withdrawal constraint may only be satisfied with $I_1 > 0$ when

$$P_1 = P_2$$

which pins down P_1 and also corresponds to

$$I_1 = (A\alpha^{\alpha})^{\frac{1}{1-\alpha}} P_2^{\frac{\alpha}{1-\alpha}}$$

as per (3.3).[102]

Note also that the no-withdrawal constraint prevents the bank from converting M_1 to risky assets in the second period as long as $\phi > \underline{\phi}$. As such, the bank may not profit from holding cash in the first period and M_1 is indeterminate. Therefore, the expected payoff can be written as

$$
\begin{aligned}
E\left[\Pi^{TB}\right] &= (1-q)(\sigma_h - P_2) I_1 \\
&= (1-q)(\sigma_h - P_2)(A\alpha^{\alpha})^{\frac{1}{1-\alpha}} P_2^{\frac{\alpha}{1-\alpha}}
\end{aligned}
$$

and its derivative with respect to P_2 is

$$\frac{\partial E\left[\Pi^{TB}\right]}{\partial P_2} = (A\alpha^{\alpha})^{\frac{1}{1-\alpha}} \frac{1-q}{1-\alpha} \left(\frac{\alpha\sigma_h}{P_2} - 1\right) P_2^{\frac{\alpha}{1-\alpha}}$$

such that

$$\frac{\partial E\left[\Pi^{TB}\right]}{\partial P_2} < 0 \;\forall\; P_2 > \alpha\sigma_h$$

C4.2 Case 2

In the second case, the no-withdrawal constraint binds but the no-short-sale constraint is slack. By combining (C56) and (C57), we can write

$$
\begin{aligned}
I_2 &= \frac{P_2 - P_1}{P_2 - \sigma_l} I_1 \\
M_2 &= P_2 \left(\frac{P_1 - \sigma_l}{P_2 - \sigma_l}\right) I_1 + M_1
\end{aligned}
\tag{C58}
$$

[102] Any solution with $I_1 = 0$ is sub-optimal as (3.3) indicates that P_1 would approach zero.

where $P_2 > \sigma_l$ follows from $\phi > \underline{\phi}$ and M_1 is indeterminate as in Case 1. Substituting these into (C55) yields the following first order condition for I_1

$$P_1 = \frac{1}{1+\mu} \frac{(1-q)\,\sigma_h\,(P_2-\sigma_l) + q\,(1-p)\,(\sigma_h-\sigma_l)\,P_2}{q\,(1-p)\,(\sigma_h-P_2) + (1-qp)\,(P_2-\sigma_l)} \tag{C59}$$

and the expected payoff is

$$\Pi = \left[(1-q)\,\sigma_h + q\,(1-p)\,(\sigma_h-\sigma_l)\,\frac{P_2}{P_2-\sigma_l}\right]\frac{\mu}{1+\mu} I_1 \tag{C60}$$

Finally, we derive a condition to eliminate this case by considering the no-short-sale constraint $I_2 \geq 0$. Since $P_2 > \sigma_l$, $I_1 > 0$, (C58) indicates that $I_2 \geq 0$ will bind when

$$P_2 < P_1$$

Using (C59), we can write this as

$$P_2 < \frac{1}{1+\mu} \frac{(1-q)\,\sigma_h\,(P_2-\sigma_l) + q\,(1-p)\,(\sigma_h-\sigma_l)\,P_2}{(1-qp)\,(P_2-\sigma_l) + q\,(1-p)\,(\sigma_h-P_2)} \tag{C61}$$

which implicitly establishes a boundary fire-sale discount $\hat{\phi}$ above which the no-short-sale constraint is slack.[103] Case 2 is eliminated for all $\phi \in [0,1]$ when $\hat{\phi} > 1$. The relevant condition can then be attained by combining (C61) with $\phi = 1$ such that

$$(1-p)\,\sigma_h + p\sigma_l < \frac{1}{1+\mu}$$
$$\therefore \mu < \bar{\mu}$$

which indicates that the mark-up restriction (3.4) eliminates Case 2.

Note also that even in the absence of the restriction (3.4), the equilibrium fire-sale never occurs under this case since the expected payoff (C60) is decreasing in ϕ. Without the restriction (3.4), the size of the shadow banking sector simply continues to expand until $\phi < \hat{\phi}$ and the equilibrium occurs under Case 1.

$$\bar{\phi} \equiv \frac{(1-q)\,\frac{\sigma_h}{1+\mu} + q\sigma_l}{(1-p)\,\sigma_h + p\sigma_l}$$

[103] With $\sigma_l = 0$, we can get an explicit expression $\hat{\phi} = \frac{1}{1-q}\left(\frac{1}{1+\mu}\frac{1-qp}{1-p} - q\right)$.

C4.3 Case 3

In the third case, the no-withdrawal constraint is slack. Due to limited liability, banks strictly prefer to convert their cash to risky assets I_2 following bad news to profit from the decline in P_2. Therefore, we can write

$$\begin{aligned} I_2 &= I_1 + \frac{M_1}{P_2} \\ M_2 &= 0 \end{aligned}$$

and the first order conditions for (M_1, P_1) are respectively written as

$$\begin{aligned} P_1 &= \frac{\sigma_h}{1+\mu} \\ P_2 &< \sigma_h \end{aligned}$$

Since $P_2 < \sigma_h$ even without a fire-sale under bad news, banks optimally hold $M_1 \to \infty$. In other words, with the no-withdrawal constrain slack, banks find it profitable to hold as much cash as possible in the first period and then convert all of it into risky assets after bad news. Since each unit of M_1 requires a unit of deposits, and risky assets contribute to low state revenues by $\sigma_l < 1$, it is impossible for the no-withdrawal constraint to remain slack under this investment strategy. Therefore, Case 3 is also eliminated.

C5 Solution under $\phi = \underline{\phi}$

Suppose $\phi < \underline{\phi}$ and hence $P_2 < \sigma_l$ such that traditional banks benefit from buying risky assets both in terms of profits and in terms of the no-withdrawal constraint. Therefore, they find it optimal to hold risky assets until the secondary market price returns to $P_2 = \sigma_l$ (i.e. $\phi = \underline{\phi}$). Let $\tilde{I}_2$ indicate the level of assets that achieve this, implicitly defined by the expression[104]

$$\frac{\sigma_l}{(1-p)\,\sigma_h + p\sigma_l} = f\left(\gamma I_1^{SB} + (1-\gamma)\left(I_1 - \tilde{I}_2\right)\right)$$

Once the secondary market price reaches $P_2 = \sigma_l$, the no-withdrawal constraint binds and traditional banks behave as described in Section 3.2. Therefore, I_1 is set according to $P_1 = P_2 = \sigma_l$ and traditional banks' holdings of safe assets in period 2 is given by

$$\begin{aligned} M_2 &= P_2 I_1 - P_2 \tilde{I}_2 \\ &= \sigma_l \left(I_1 - \tilde{I}_2\right) \end{aligned}$$

[104] We drop the label 'TB' to simplify the exposition

where we have taken advantage of the indeterminacy of $M_1 \geq 0$ to set $M_1 = 0$. Finally, expected profits are given by

$$
\begin{aligned}
E[\Pi] &= (\sigma_h - \sigma_l)\left[(1-q) I_1 + q(1-p)\tilde{I}_2\right] \\
&= (\sigma_h - \sigma_l)\left[(1-q) A^{\frac{1}{1-\alpha}} (\alpha\sigma_l)^{\frac{\alpha}{1-\alpha}} + q(1-p)\tilde{I}_2\right]
\end{aligned}
$$

So far, we have assumed that traditional banks remain net-sellers with $\tilde{I}_2 \leq I_1$. When the excess supply of assets is particularly large, we may have $P_2 < \sigma_l$ even when traditional banks hold on to their risky assets such that $\tilde{I}_2 = I_1$. In this case, they will find it optimal to increase D and M_1 use this to purchase risky assets in period 2 until $P_2 = \sigma_l$. As before, the no-withdrawal constraint will bind at $P_2 = \sigma_l$ and the complete solution is

$$
\begin{aligned}
P_1 &= P_2 \\
I_2 &= I_1 + \frac{1}{\sigma_l}\tilde{M}_1 \\
M_2 &= 0 \\
D &= P_1 I_1 + \tilde{M}_1 = \sigma_l I_1 + \tilde{M}_1
\end{aligned}
$$

where $\tilde{M}_1$ takes on the role of ensuring $P_2 = \sigma_l$ and is implicitly defined by

$$
\frac{\sigma_l}{(1-p)\sigma_h + p\sigma_l} = f\left(\gamma I_1^{SB} - (1-\gamma)\frac{1}{\sigma_l}\tilde{M}_1\right)
$$

and the expected payoff is

$$
\begin{aligned}
E[\Pi] &= (\sigma_h - \sigma_l)\left((1-qp) A^{\frac{1}{1-\alpha}} (\alpha\sigma_l)^{\frac{\alpha}{1-\alpha}} + q(1-p)\frac{1}{\sigma_l}\tilde{M}_1\right) \\
&= (\sigma_h - 1)\left(\frac{1-qp}{qp} A^{\frac{1}{1-\alpha}} \left(\alpha\frac{1-(1-qp)\sigma_h}{qp}\right)^{\frac{\alpha}{1-\alpha}} + \frac{q(1-p)}{1-(1-qp)\sigma_h}\tilde{M}_1\right)
\end{aligned}
$$

There are two notable implications. First, the secondary market price cannot go below σ_l. Second, a rise in the shadow banking sector size γ first leads to a rise in $\tilde{I}_2$, and then $\tilde{M}_1$. The above solution shows that $\tilde{M}_1$ and $E[\Pi]$ rises in this case while everything else stays constant. As we move to a limiting case with only shadow banks, safe asset holdings and traditional bank profits both approach infinity

$$
\lim_{\gamma \to 1} E\left[\Pi^{TB}\right] = \lim_{\gamma \to 1} \tilde{M}_1 = \infty
$$

which guarantees an inferior equilibrium for a sufficiently high commitment cost $\tau > \bar{\tau}$.

C6 Proof for Proposition 3.2

For the purposes of the proof, it is convenient to introduce some additional notation. Let ϕ^* denote the equilibrium fire-sale discount and the functions $\left(\pi^{SB}(.), \pi^{TB}(.)\right)$ map from the fire-sale discount to expected payoffs from shadow and traditional banking such that

$$\begin{aligned}\pi^{SB}(\phi) &= (1-q)\frac{\mu}{1+\mu}\sigma_h (A\alpha^\alpha)^{\frac{1}{1-\alpha}}\left((1-q)\frac{\sigma_h}{1+\mu}+q\phi\left[(1-p)\sigma_h+p\sigma_l\right]\right)^{\frac{\alpha}{1-\alpha}}\\ \pi^{TB}(\phi) &= (1-q)\left(\sigma_h-\phi\left[(1-p)\sigma_h+p\sigma_l\right]\right)(A\alpha^\alpha)^{\frac{1}{1-\alpha}}\left(\phi\left[(1-p)\sigma_h+p\sigma_l\right]\right)^{\frac{\alpha}{1-\alpha}}\end{aligned}$$

as per Lemma 3.1 and Lemma 3.2. There is an interior equilibrium when the following sufficient conditions are satisfied

$$\pi^{SB}\left(\overline{\phi}\right) > \pi^{TB}\left(\overline{\phi}\right)-\tau \tag{C62}$$

$$\pi^{SB}\left(\underline{\phi}\right) > \pi^{TB}\left(\underline{\phi}\right)-\tau \tag{C63}$$

$$\frac{\partial\pi^{SB}(\phi)}{\partial\phi} > \frac{\partial\pi^{TB}(\phi)}{\partial\phi}\ \forall\ \phi\in(\phi^*,1) \tag{C64}$$

$$\frac{\partial\phi}{\partial\gamma} < 0\ \forall\ \gamma\in[0,1] \tag{C65}$$

where $\tau > 0$. In the sections below, we show that these conditions will be satisfied within a range of commitment costs $\tau\in(\underline{\tau},\bar{\tau})$ and also show that this range is non-empty.

C6.1 Proof for condition (C62)

The condition depends on the effective value taken by

$$\bar{\phi}\equiv\min\left[1,\frac{(1-q)\frac{\sigma_h}{1+\mu}+q\sigma_l}{(1-p)\sigma_h+p\sigma_l}\right]$$

When we have

$$\mu<\frac{(p-q)(\sigma_h-1)}{qp(1-q)\sigma_h-(p-q)(\sigma_h-1)} \tag{C66}$$

such that $\bar{\phi}=1$, the relevant condition is

$$\pi^{SB}(1)>\pi^{TB}(1)-\tau$$

Using the definitions for $\left(\pi^{SB}\left(.\right), \pi^{TB}\left(.\right)\right)$, we can write this condition as a minimum commitment cost

$$\tau \geq \underline{\tau} \equiv (1-q)\left(A\alpha^{\alpha}\right)^{\frac{1}{1-\alpha}}(\sigma_h - 1)\left(\frac{1}{q}\right)^{\frac{1}{1-\alpha}}\left(1-(1-q)\,\sigma_h\right)^{\frac{\alpha}{1-\alpha}}$$
$$-(1-q)\left(A\alpha^{\alpha}\right)^{\frac{1}{1-\alpha}}\frac{\mu}{1+\mu}\sigma_h\left(1-\frac{\mu}{1+\mu}(1-q)\,\sigma_h\right)^{\frac{\alpha}{1-\alpha}} > 0$$

As an aside, we also show that $\underline{\tau} > 0$ such that a commitment cost is necessary for an interior equilibrium. To do this, note that the expected payoff under Case 1 and Case 2 of the traditional bank's problem in Appendix C4 are equivalent when $\phi = 1$, $\mu = \bar{\mu}$. For any $\mu < \bar{\mu}$, profits under Case 1 are higher. Therefore, we can write a sufficient condition

$$\begin{aligned}
\left(A\alpha^{\alpha}\right)^{\frac{1}{1-\alpha}}\frac{\bar{\mu}}{1+\bar{\mu}}\left(\frac{1}{1+\bar{\mu}}\right)^{\frac{\alpha}{1-\alpha}} &> \left(A\alpha^{\alpha}\right)^{\frac{1}{1-\alpha}}\frac{\bar{\mu}}{1+\bar{\mu}}(1-q)\,\sigma_h\left(1-\frac{\bar{\mu}}{1+\bar{\mu}}(1-q)\,\sigma_h\right)^{\frac{\alpha}{1-\alpha}} \\
\therefore \quad & 1 > (1-q)\,\sigma_h\left(1+\bar{\mu}\left(1-(1-q)\,\sigma_h\right)\right)^{\frac{\alpha}{1-\alpha}}
\end{aligned}$$

Since $\frac{\alpha}{1-\alpha} < 1$ for $\alpha < \frac{1}{2}$, we can eliminate the power and substitute for $\bar{\mu} = \frac{(1-q)(\sigma_h-1)}{1-(1-q)\sigma_h}$ to get a sufficient condition

$$(1-q)\,\sigma_h\left[(1-q)\,\sigma_h + q\right] < 1$$

Note that the RHS is increasing in σ_h. A further sufficient condition is then to set $\sigma_l = 0$ which maximizes σ_h. We can then see that the above condition is true for all $p < 1$. Therefore, we can show that $\tau > 0$ under the two conditions

$$\begin{aligned}
\alpha &< \frac{1}{2} \\
\mu &\leq \bar{\mu}
\end{aligned}$$

When (C66) is not satisfied such that $\bar{\phi} < 1$, the relevant condition for (C62) is

$$\pi^{SB}\left(\bar{\phi}\right) > \pi^{TB}\left(\bar{\phi}\right) - \tau$$

which leads to a higher minimum commitment cost

$$\begin{aligned}
\underline{\tau} = {} & (1-q)\left(A\alpha^{\alpha}\right)^{\frac{1}{1-\alpha}}\left(\sigma_h - \bar{\phi}\left[(1-p)\,\sigma_h + p\sigma_l\right]\right)\left(\bar{\phi}\left[(1-p)\,\sigma_h + p\sigma_l\right]\right)^{\frac{\alpha}{1-\alpha}} - \\
& (1-q)\left(A\alpha^{\alpha}\right)^{\frac{1}{1-\alpha}}\frac{\mu}{1+\mu}\sigma_h\left((1-q)\,\frac{\sigma_h}{1+\mu} + q\bar{\phi}\left[(1-p)\,\sigma_h + p\sigma_l\right]\right)^{\frac{\alpha}{1-\alpha}}
\end{aligned}$$

where the aside on $\tau > 0$ is still valid.

C6.2 Proof for condition (C63)

It follows from Appendix C5 that (C63) will be satisfied when the lower bound restriction on ϕ is violated such that

$$\underline{\phi} > f\left((A\alpha^{\alpha})^{\frac{1}{1-\alpha}} \left((1-q)\frac{\sigma_h}{1+\mu} + q\sigma_l \right)^{\frac{\alpha}{1-\alpha}} \right)$$

for any $\tau < \infty$. Therefore, we do not necessarily need an upper bound on the commitment cost for an interior equilibrium. However, the interior equilibrium has different properties in the region $\phi < \underline{\phi}$ (as described in Appendix C5) and we impose an upper bound on the commitment cost to prevent this.

Let $\widetilde{\phi} \equiv \frac{\alpha\sigma_h}{(1-p)\sigma_h + p\sigma_l}$ denote the fire-sale discount that maximizes traditional bank profits. The upper bound depends on where $\widetilde{\phi}$ stands relative to $\underline{\phi}$. When the following condition is true

$$\sigma_h \geq \frac{1}{1 - qp\,(1-\alpha)} \tag{C67}$$

such that $\widetilde{\phi} > \underline{\phi}$, the upper bound for commitment costs $\bar{\tau}$ must satisfy

$$\pi^{SB}\left(\widetilde{\phi}\right) < \pi^{TB}\left(\widetilde{\phi}\right) - \tau$$

which yields the upper bound

$$\tau \leq \bar{\tau} = (1-q)\,(\sigma_h A\alpha^{\alpha})^{\frac{1}{1-\alpha}} \left[(1-\alpha)\,\alpha^{\frac{\alpha}{1-\alpha}} - \frac{\mu}{1+\mu}\left(\frac{1-q}{1+\mu} + q\alpha\right)^{\frac{\alpha}{1-\alpha}} \right]$$

When (C67) is not satisfied, the fire-sale discount hits $\phi = \underline{\phi}$ before traditional bank profits peak and the upper bound is given by

$$\tau \leq \bar{\tau} = (1-q)\,(A\alpha^{\alpha})^{\frac{1}{1-\alpha}} \left[(\sigma_h - \sigma_l)\,\sigma_l^{\frac{\alpha}{1-\alpha}} - \frac{\mu}{1+\mu}\sigma_h \left((1-q)\frac{\sigma_h}{1+\mu} + q\sigma_l \right)^{\frac{\alpha}{1-\alpha}} \right]$$

Note that, regardless of the value taken by $(\bar{\tau}, \underline{\tau})$, it follows from (C64) that $\bar{\tau} > \underline{\tau}$.

C6.3 Proof for condition (C64)

This follows directly from Lemma 3.1, which shows that

$$\frac{\partial \pi^{SB}(\phi)}{\partial \phi} > 0 \;\forall\; \phi \in (0,1)$$

and Lemma 3.2 which shows that

$$\frac{\partial \pi^{TB}(\phi)}{\partial \phi} < 0 \ \forall \ \phi \in \left(\widetilde{\phi}, \bar{\phi}\right)$$

where $\phi^* > \widetilde{\phi}$ since the latter is the peak of traditional bank profits in the range above $\underline{\phi}$.

C6.4 Proof for condition (C65)

Recall that the excess supply of assets is given by

$$\widetilde{I} = \gamma I_1^{SB} + (1-\gamma)\left(I_1^{TB} - I_2^{TB}\right) > 0$$

where $\left(I_1^{SB}, I_1^{TB}\right)$ depend on ϕ and $I_2^{TB} = 0$. Given that $f(.)$ is continuous and decreasing, to satisfy (C65) we require that

$$\frac{\partial \widetilde{I}}{\partial \gamma} = I_1^{SB} - I_1^{TB} > 0 \ \forall \ \gamma \in [0,1]$$

which is equivalent to

$$I_1^{SB} > I_1^{TB} \ \forall \phi \in (\underline{\phi}, \overline{\phi}]$$

At any given ϕ, we have $I_1^{SB} > I_1^{TB}$ when the following condition is satisfied

$$q\sigma_h > (1+\mu)\,\phi\,[1-(1-q)\,\sigma_h]$$

Since the RHS is increasing in μ and ϕ, a sufficient condition is to set $\phi = 1$, $\mu = \bar{\mu}$, which will be satisfied for $\sigma_h > 1$.

Since $\left(I_1^{SB}, I_1^{TB}\right)$ both decrease in ϕ at different rates, we also need to show that I_1^{SB} at $\underline{\phi}$ is lower than I_1^{TB} at $\bar{\phi}$. This will be true with $\bar{\phi} = 1$ when (C66) is satisfied. Otherwise, $\bar{\phi}$ will need to satisfy

$$\bar{\phi} = \frac{(1-q)\frac{\sigma_h}{1+\mu} + q\sigma_l}{(1-p)\,\sigma_h + p\sigma_l}$$

which is precisely how we define the upper bound restriction on the fire-sale discount.

C6.5 Proof for the non-emptiness of $(\underline{\tau}, \bar{\tau})$

Finally, we prove that $\bar{\tau} > \underline{\tau}$ such that there is a non-empty set of commitment costs that bring about an interior equilibrium. Since there are two alternatives values for both $\bar{\tau}$ and $\underline{\tau}$, we consider each in turn. First suppose that (C66) is satisfied so that $\underline{\tau}$ is in line with $\phi = 1$. Then it follows from Lemma 3.2 that $\bar{\tau} > \underline{\tau}$ regardless of which value $\bar{\tau}$ takes. Second, suppose

(C66) is not satisfied so that $\underline{\tau}$ is in line with $\bar{\phi} < 1$. When (C67) is also not satisfied such that traditional bank profits do not peak until $\underline{\phi}$, $\bar{\tau} > \underline{\tau}$ follows from $\bar{\phi} > \underline{\phi}$.

The only case where we need impose an additional condition corresponds to (C67) being satisfied so that $\bar{\tau}$ is in line with the peak $\phi = \frac{\alpha\sigma_h}{(1-p)\sigma_h + p\sigma_l}$ while (C66) is not satisfied such that $\bar{\phi} < 1$. The condition for non-emptiness is then equivalent to

$$\widetilde{\phi} < \bar{\phi}$$

$$\therefore \frac{\alpha\sigma_h}{(1-p)\,\sigma_h + p\sigma_l} < \frac{(1-q)\,\frac{\sigma_h}{1+\mu} + q\sigma_l}{(1-p)\,\sigma_h + p\sigma_l}$$

A sufficient condition is

$$\alpha < 1 - p$$

which should be satisfied when $\alpha < 0.5$, $p \leq 0.5$.

C7 Proof for Proposition 3.4.3

We first solve the household's problem to attain the expressions (3.28), (3.29). Allowing for liquidity risk, the household's problem can be written as

$$\max_{d^{SB},d^{TB},M_1,M_2} \quad (1-q)\,C_{gh} + q\,(1-p)\,C_{bh} + qpC_{bl}$$

$$\text{s.t.}$$

$$d^{SB} + d^{TB} + M_1 = E$$

$$M_2 = M_1 + \theta^{SB} d^{SB}$$

$$C_{gh} = M_1 + d^{TB} R^{TB} + d^{SB} R^{SB}$$

$$C_{bh} = M_2 + \left[(1-\xi)\,R^{TB} + \xi\theta^{TB}\right] d^{TB}$$

$$C_{bl} = M_2 + \left[(1-\xi)\,\bar{V} R^{TB} + \xi\theta^{TB}\right] d^{TB}$$

with the first order conditions (3.14) for deposits in shadow banks and

$$R^{TB} = 1 + \frac{q\xi\left(1-\theta^{TB}\right) + qp\,(1-\xi)\left(1-\bar{V}\right)}{1 - q\left(\xi + (1-\xi)\,p\left(1-\bar{V}\right)\right)} \tag{C68}$$

for deposits in traditional banks.[105]

[105] Under the sequential service constraint, a portion θ^{TB} of depositors are able to withdraw all of their funds during a bank-run while the remainder receive no payment. We assume that the outcome of an attempted withdrawal is idiosyncratic to the household-bank pairing and that households diversify their holdings across a large number of traditional banks. This does not affect our results but streamlines the exposition by preventing households from becoming heterogenous in whether they have been repaid.

Early withdrawal decision and market discipline

We assume that the bank-run and the early withdrawal decision take place simultaneously such that, realistically, households may not secure themselves from bank-runs with an early withdrawal. Expected consumption without and with an early withdrawal are then respectively given by

$$\begin{aligned}(1-P)\,c_{bh}+pc_{bl} &= M_1+\theta^{SB}d^{SB}+\left[\left(1-p\left(1-\bar{V}\right)\right)(1-\xi)\,R^{TB}+\xi\theta^{TB}\right]d^{TB}\\ c_b^w &= M_1+\theta^{SB}d^{SB}+\left(1-\left(1-\theta^{TB}\right)\xi\right)d^{TB}\end{aligned}$$

where the latter expression indicates that a household that decides to withdraw its deposit early may receive an incomplete repayment due to a bank-run. Under this set up, all terms that relate to the bank-run probability cancel out and the minimum repayment rate schedule is given by

$$\begin{aligned}(1-P)\,c_{bh}+pc_{bd} &\geq c_b^w\\ \therefore V\geq\bar{V}\equiv\frac{1}{p}&\left(\frac{1}{R^{TB}}-(1-p)\right)\end{aligned}\tag{C69}$$

which remains identical to the simple model. Combining (C68) and (C69) then yields

$$\begin{aligned}\bar{V} &= 1-\frac{q}{p}\frac{\xi\left(1-\theta^{TB}\right)}{1-q\left(1-\xi\left(1-\theta^{TB}\right)\right)}\\ R^{TB} &= 1+\frac{q}{1-q}\xi\left(1-\theta^{TB}\right)\end{aligned}$$

With $\xi=0$, these expressions simplify to $\bar{V}=R^{TB}=1$. Note that $\bar{V}$ is decreasing in ξ and R^{TB} is increasing. Therefore a rise in ξ leads to

$$\bar{V}<1<R^{TB}$$

Fire-sale on the safe asset and liquidity shortfall

Finally, we show that the fire-sale on the safe asset is a crucial determinant of liquidity risk. Consider the liquidation value given by (3.31). There will be no liquidity shortfall such that $\theta^{TB}=1$ under the condition

$$P_2\,(r)\,I_2^{TB}\,(r)+P_2\,(s)\,I_2^{TB}\,(s)\geq D^{TB}$$

First, consider the case without a fire-sale on safe assets such that $P_2(s) = 1$. The condition becomes

$$I_2^{TB}(r) + I_2^{TB}(s) \geq D^{TB}$$

and will be true under any investment strategy that satisfies the no-withdrawal constraint (3.30) as long as

$$P_2(r) \geq \sigma_l$$

To see that this must be true, consider what would happen otherwise. Since traditional banks are protected by limited liability, they do not internalize the state with weak fundamentals. Therefore, given $P_2(s) = 1 \geq P_2(r)$, traditional banks always prefer to purchase risky assets which yield a higher return in the state where they remain solvent. Ordinarily, traditional banks' risky asset purchases are limited by the no-withdrawal constraint. However, with $P_2(r) < \sigma_l$ and $P_2(s) = 1$, the strategy of selling a safe asset and purchasing risky assets with the funds increases the recovery rate V. Consequently, traditional banks increase their purchases of risky assets until their price rises to $P_2(r) = \sigma_l$.

Second, consider the case with a fire-sale on safe assets such that $P_2(s) = \phi$. Since shadow banks are liquidated after bad news, and traditional banks re-allocate their portfolio from risky to safe assets, there is an excess supply of risky assets at all times such that $P_2(r) = \phi$. We can then write the condition for $\theta^{TB} = 1$ as

$$\phi\left(I_2^{TB}(r) + I_2^{TB}(s)\right) \geq D^{TB} \tag{C70}$$

Note that the no-withdrawal constraint is not tightened by a decline in ϕ when both safe and risky assets are in excess supply, since the terms of trade between the two assets do not change, while the value of liquid asset holdings increase. Therefore, for sufficiently low ϕ, (C70) fails and there is a liquidity shortfall $\theta^{TB} < 1$.

C8 Example fire-sale function

To attain a simple fire-sale function from the outsider investor's problem, we can simply parameterize the payoff function from the outside investment to

$$g\left(\widetilde{K}\right) = z^{-1}\ln\left(\widetilde{K}\right)$$

where $z > 0$. The fire-sale function then becomes

$$f\left(\widetilde{I}\right) = \frac{z\widetilde{E}}{1 + z\left[(1-p)\sigma_h + p\sigma_l\right]\widetilde{I}}$$

To satisfy the lower bound condition exactly, we need to set $\widetilde{E}$ at a level that yields $f\left(I_1^{SB}\right) = \underline{\phi}$ at $\gamma = 0$, which is

$$\widetilde{E} = \underline{\phi}\left[\frac{1}{z} + (A\alpha^{\alpha})^{\frac{1}{1-\alpha}}\left((1-q)\frac{\sigma_h}{1+\mu} + \frac{1-(1-qp)\sigma_h}{p}\right)^{\frac{\alpha}{1-\alpha}}\right]$$

and the upper bound will approach but never exceed $\bar{\phi}$ as z rises.

C9 Full description of the model with liquidity risk

We describe banks and entrepreneurs below. Outside investors are described in Section 3.4.2 while households are described under Appendix C7.

C9.1 Entrepreneurs

Entrepreneurs only differ from those described in Section 3.3.1.1 in that they may produce all three asset types $i \in \{\lambda, s, r\}$. We assume that they have a Cobb-Douglas production function that is additively separable in the asset type such that

$$I_1(i) = AK(i)^{\alpha} \;\; \forall i \in \{\lambda, s, r\}$$

where $K(i)$ is an investment in capital specific to the asset type. This yields the set of first order conditions

$$P_1(i) = \frac{1}{\alpha A^{\frac{1}{\alpha}}} I_1(i)^{\frac{1-\alpha}{\alpha}} \;\; \forall i \in \{\lambda, s, r\} \tag{C71}$$

where we impose a constant markup μ across assets for simplicity.

C9.2 Banks

With a richer asset space, the first period budget constraint becomes

$$\sum_{i \in \{\lambda, s, r\}} P_1(i) I_1(i) = D$$

where $I_1(i)$ is the amount purchased of an asset i and $P_1(i)$ is the corresponding asset price. As in the simple model, assets are priced at their expected payoff in the secondary market after good news and trade is inconsequential. Bank profits (in the third period) are then given by

$$\Pi_{gh} = \sigma_h I_1(r) + I_1(s) + I_1(\lambda) - DR \tag{C72}$$

Following bad news, banks face the second period budget constraint

$$\sum_{i\in\{s,r\}} \left(P_2\left(i\right)-P_1\left(i\right)\right)I_1\left(i\right)=I_1\left(\lambda\right) \tag{C73}$$

and have a liquidation value

$$\theta=\min\left\{1,\frac{P_2\left(r\right)I_2\left(r\right)+P_2\left(s\right)I_2\left(s\right)}{D}\right\} \tag{C74}$$

When risky assets yield a high payoff σ_h in the third period, banks make a profit

$$\Pi_{gh}=\sigma_h I_2\left(r\right)+I_2\left(s\right)-DR$$

while limited liability binds under a low asset payoff σ_l. Under limited liability, banks make zero profits and deposits pay a recovery rate

$$V=\min\left\{1,\frac{\sigma_l I_2\left(r\right)+I_2\left(s\right)}{DR}\right\}$$

which is proportional to the shortfall of funds. We can use this expression to write the no-withdrawal constraint for traditional banks as

$$\sigma_l I_2^{TB}\left(r\right)+I_2^{TB}\left(s\right)\geq\bar{V}D^{TB}R^{TB} \tag{C75}$$

Next, we evaluate the optimal behaviour of banks under the two alternative strategies of shadow and traditional banking. The free entry condition is given by (3.26) as in the simple model.

Shadow banking

Shadow banks choose $\left\{I_1^{SB}\left(i\right),D^{SB},i\in\{\lambda,s,r\}\right\}$ to maximize their expected profits

$$E\left[\Pi^{SB}\right]=(1-q)\left(\sigma_h I_1^{SB}\left(r\right)+I_1^{SB}\left(s\right)+I_1^{SB}\left(\lambda\right)-D^{SB}R^{SB}\right)$$

subject to (C72). The optimal portfolio allocation is then determined by the set of first order conditions

$$\begin{aligned}P_1\left(r\right)&=\frac{\sigma_h}{\left(1+\mu\right)R^{SB}}\\P_1\left(s\right)&=P_1\left(\lambda\right)=\frac{1}{\left(1+\mu\right)R^{SB}}\end{aligned}$$

which allow us to back out the asset holdings in period 1 using (C71).

Traditional banking

Traditional banks choose $\left\{I_1^{TB}(i), I_2^{TB}(r), I_2^{TB}(s), M_1^{TB}, M_2^{TB}, D^{TB}, i \in \{\lambda, s, r\}\right\}$ to maximize their expected profits

$$\begin{aligned} E\left[\Pi^{TB}\right] &= (1-q)\left(\sigma_h I_1^{TB}(r) + I_1^{TB}(s) + I_1^{TB}(\lambda)\right) \\ &\quad + q(1-p)(1-\xi)\left(\sigma_h I_2^{TB}(r) + I_2^{TB}(s)\right) \\ &\quad - (1-q+q(1-p)(1-\xi)) D^{TB} R^{TB} \end{aligned}$$

subject to (C72), (C73) and (C75). Due to their ability to commit, traditional banks also internalize the relationship between their liquidation value θ^{TB} given by (C74) and the minimum recovery rate $\bar{V}$ and R^{TB} as per the expressions in Proposition 3.3.

We focus on the case with a slack no-short-sale constraint

$$I_1^{TB}(r) > 0$$

which is the case presented in our results. By combining (C73) and (C75), we can write the following expressions for second period asset holdings

$$\begin{aligned} I_2^{TB}(s) &= \frac{P_2^{TB}(r)\bar{V}R^{TB}D^{TB} - \sigma_l\left(P_2^{TB}(s)I_1^{TB}(s) + P_2^{TB}(r)I_1^{TB}(r) + I_1^{TB}(\lambda)\right)}{P_2^{TB}(r) - \sigma_l P_2^{TB}(s)} \\ I_2^{TB}(r) &= \frac{P_2^{TB}(s)\left(I_1^{TB}(s) - \bar{V}R^{TB}D^{TB}\right) + P_2^{TB}(r)I_1^{TB}(r) + I_1^{TB}(\lambda)}{P_2^{TB}(r) - \sigma_l P_2^{TB}(s)} \end{aligned}$$

When there is a liquidity shortfall $\theta^{TB} < 1$, we can use the expressions in Proposition 3.3 and (C74) to write the problem as

$$\begin{aligned} E\left[\Pi^{TB}\right] &= (1-q)\left(\sigma_h I_1^{TB}(r) + I_1^{TB}(s) + I_1^{TB}(\lambda)\right) \\ &\quad + \left[P_2^{TB}(s)I_1^{TB}(s) + P_2^{TB}(r)I_1^{TB}(r) + I_1^{TB}(\lambda)\right] * \left[q\xi\frac{1-q+q(1-p)(1-\xi)}{1-q}\right. \\ &\quad \left. + q\frac{(1-p)(1-\xi)}{P_2^{TB}(r) - \sigma_l P_2^{TB}(s)}\left((\sigma_h - \sigma_l) - \xi\frac{q(1-p)}{p(1-q)}\left(\sigma_h P_2^{TB}(s) - P_2^{TB}(r)\right)\right)\right] \\ &\quad - (1-q+q(1-p)(1-\xi))\left(1 + \frac{q}{1-q}\xi\right) D^{TB} \\ &\quad - q(1-p)(1-\xi)\left(1 - \xi\frac{q(1-p)}{p(1-q)}\right)\left(\frac{\sigma_h P_2^{TB}(s) - P_2^{TB}(r)}{P_2^{TB}(r) - \sigma_l P_2^{TB}(s)}\right) D^{TB} \end{aligned}$$

s.t.

$$P_1^{TB}(\lambda)I_1^{TB}(\lambda) + P_1^{TB}(s)I_1^{TB}(s) + P_1^{TB}(r)I_1^{TB}(r) = D^{TB}$$

which yields the first order conditions

$$
\begin{aligned}
P_1^{TB}(r) &= \frac{1}{1+\mu}\frac{(1-q)\sigma_h + \bar{Z}_1 P_2^{TB}(r)}{\bar{Z}_2} \\
P_1^{TB}(s) &= \frac{1}{1+\mu}\frac{1-q+\bar{Z}_1 P_2^{TB}(s)}{\bar{Z}_2} \\
P_1^{TB}(\lambda) &= \frac{1}{1+\mu}\frac{1-q+q\bar{Z}}{\bar{Z}_2}
\end{aligned}
$$

where

$$
\begin{aligned}
\bar{Z}_1 &\equiv q\frac{\xi\left(1-q+q(1-p)(1-\xi)\right)}{1-q} \\
&\quad + \frac{q(1-p)(1-\xi)}{P_2^{TB}(r)-\sigma_l P_2^{TB}(s)}\left((\sigma_h-\sigma_l)-\xi\frac{q(1-p)}{p(1-q)}\left(\sigma_h P_2^{TB}(s)-P_2^{TB}(r)\right)\right) \\
\bar{Z}_2 &\equiv \left(1-q+q(1-p)(1-\xi)\right)\left(1+\frac{q}{1-q}\xi\right) \\
&\quad + q(1-p)(1-\xi)\left(1-\xi\frac{q(1-p)}{p(1-q)}\right)\left(\frac{\sigma_h P_2^{TB}(s)-P_2^{TB}(r)}{P_2^{TB}(r)-\sigma_l P_2^{TB}(s)}\right)
\end{aligned}
$$

When there is no liquidity shortfall $\theta^{TB}=1$, we can use $R^{TB}=\bar{V}=1$ from Proposition 3.3 to write the problem as

$$
\begin{aligned}
E\left[\Pi^{TB}\right] &= (1-q)\left(\sigma_h I_1^{TB}(r)+I_1^{TB}(s)+I_1^{TB}(\lambda)\right) \\
&\quad + q(1-p)(\sigma_h-\sigma_l)\frac{P_2^{TB}(s)I_1^{TB}(s)+P_2^{TB}(r)I_1^{TB}(r)+I_1^{TB}(\lambda)}{P_2^{TB}(r)-\sigma_l P_2^{TB}(s)} \\
&\quad - \left(q(1-p)\left(\frac{\sigma_h P_2^{TB}(s)-P_2^{TB}(r)}{P_2^{TB}(r)-\sigma_l P_2^{TB}(s)}\right)+(1-qp)\right)D^{TB}
\end{aligned}
$$

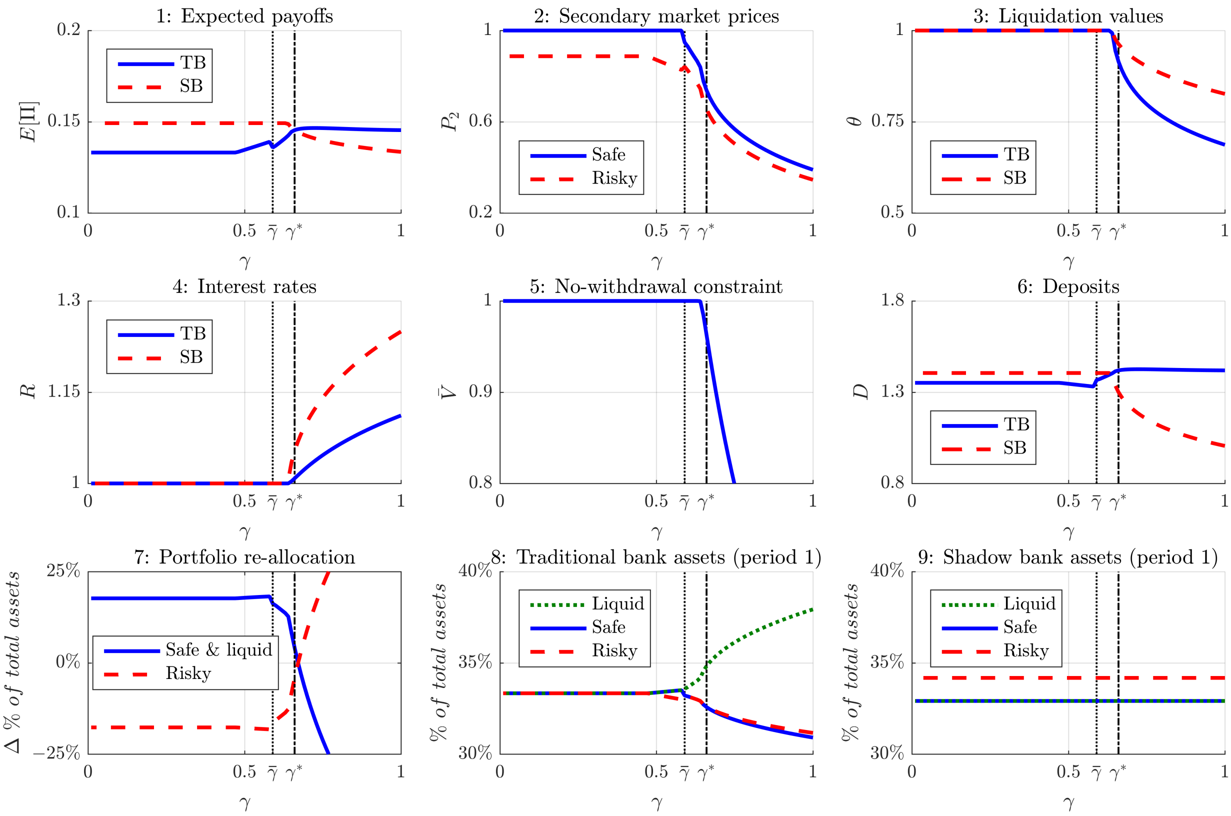
s.t.

$$
P_1^{TB}(\lambda)I_1^{TB}(\lambda)+P_1^{TB}(s)I_1^{TB}(s)+P_1^{TB}(r)I_1^{TB}(r)=D^{TB}
$$

The first order conditions FOCs then become

$$
\begin{aligned}
P_1^{TB}(r) &= \frac{1}{1+\mu}\frac{(1-q)\sigma_h\left(P_2^{TB}(r)-\sigma_l P_2^{TB}(s)\right)+q(1-p)(\sigma_h-\sigma_l)P_2^{TB}(r)}{(1-q)P_2^{TB}(r)+\left(q(1-p)\sigma_h-(1-qp)\sigma_l\right)P_2^{TB}(s)} \\
P_1^{TB}(s) &= \frac{1}{1+\mu}\frac{(1-q)\left(P_2^{TB}(r)-\sigma_l P_2^{TB}(s)\right)+q(1-p)(\sigma_h-\sigma_l)P_2^{TB}(s)}{(1-q)P_2^{TB}(r)+\left(q(1-p)\sigma_h-(1-qp)\sigma_l\right)P_2^{TB}(s)} \\
P_1^{TB}(\lambda) &= \frac{1}{1+\mu}\frac{(1-q)\left(P_2^{TB}(r)-\sigma_l P_2^{TB}(s)\right)+q(1-p)(\sigma_h-\sigma_l)}{(1-q)P_2^{TB}(r)+\left(q(1-p)\sigma_h-(1-qp)\sigma_l\right)P_2^{TB}(s)}
\end{aligned}
$$

Figure C1 : **Results under alternative bank-run specification**



Note: Expected payoffs are inclusive of the commitment cost τ. Total assets in period 1 and 2 are respectively defined as $\bar{I}_1 \equiv \sum_{i\in\{\lambda,s,r\}} I_1(i)$ and $\bar{I}_2 \equiv \sum_{i\in\{s,r\}} P_2(i)\, I_2(i)$. Panel 7 plots $\left(\bar{I}_2^{TB}\right)^{-1} P_2(s)\, I_2^{TB}(s) - \left(\bar{I}_1^{TB}\right)^{-1} \sum_{i\in\{\lambda,s\}} I_1^{TB}(i)$ for safe and liquid assets, and $\left(\bar{I}_2^{TB}\right)^{-1} P_2(r)\, I_2^{TB}(r) - \left(\bar{I}_1^{TB}\right)^{-1} I_1^{TB}(r)$ for risky assets. Panel 8 plots $I_1^{TB}(i)/\bar{I}_1$ respectively for $i = \{\lambda, s, r\}$ and Panel 9 does the same for shadow banks.

C10 Alternative specification for bank-runs

Following the global games solution of Goldstein and Pauzner (2005), we postulate that banks with a shortfall of liquidity are more vulnerable to bank-runs. Specifically, we depict the probability ξ that a bank faces a self-fulfilling run as a negative function $\zeta(.)$ of its liquidation value θ such that

$$\xi = \zeta(\theta),$$
$$\zeta'(.) \leq 0,\ \zeta(\theta) \in [0,1]\ \forall\ \theta$$

where $\zeta(1) = 0$ ensures that banks without a liquidity shortfall are not vulnerable to self-fulfilling runs.[106] We parameterize $\zeta(.)$ simply as

$$\zeta(\theta) = \max\left\{0, \min\left\{1, \widetilde{\xi}(1-\theta)\right\}\right\}$$

with $\widetilde{\xi} = 1.64$ calibrated in line with the calibration strategy described in Section 3.5.1. Figure C1 provides the numerical results under a set up and calibration that are otherwise identical to those presented in Section 3.5.2. In equilibrium, the two bank run specifications yield exactly the same outcome. At above equilibrium sizes of shadow banking ($\gamma > \gamma^*$), the alternative bank run specification implies further increases in ξ in line with the decrease in liquidity. This leads to lower traditional bank profits and a sharper decline in minimum recovery rate $\bar{V}$ compared to the baseline case.

[106] We also impose $\psi(\theta) \in [0,1]\ \forall\ \theta$ since ξ is a probability.